*HW assignment date: **01 April 2020***

Homework - I
KSU / Math-244 / Semester-II (1440-1441)

Note 1: The students must submit the completed homework through email to the respective class teachers within 3 days from its assignment date.

Note 2: Every student must submit PDF file of the homework done; in his own handwriting using blue ink and with his signature.

Problem1:

a) If $\mathbf{A}=\begin{bmatrix}1&0&1\\2&-1&1\\1&3&2\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}1&0&-3\\-6&7&-3\\-3&-9&-2\end{bmatrix}$, then **find** the values of

a. $x\,,y$ and z such that $x\mathbf{A}^2+y\,\mathbf{AB}+z\,\mathbf{I}=\mathbf{O}$.

b) **Find** $adj\,(\mathbf{A}\,)$ and $\mathbf{A}^{-1}$ for the matrix:

$$\mathbf{A}=\begin{bmatrix}1&0&a\\2&b&c\\-1&1&1\end{bmatrix},$$

where $ab+b+2a-c\neq 0$

c) Let $\mathbf{A}\in \mathbf{M}_{3\times 3}(\mathbb{R})$ with determinant $|\mathbf{A}|=2$. **Find** $|2(adj(\mathbf{A}))^{-1}+\mathbf{A}|$.

Problem 2:

a) Let $\mathbf{A}=\begin{bmatrix}\mathbf{1}&\mathbf{-1}&\mathbf{4}&\mathbf{5}\\\mathbf{-2}&\mathbf{1}&\mathbf{-11}&\mathbf{-8}\\\mathbf{-1}&\mathbf{2}&\mathbf{2}&\mathbf{2}\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}\mathbf{1}&\mathbf{-1}&\mathbf{3}&\mathbf{2}\\\mathbf{0}&\mathbf{1}&\mathbf{4}&\mathbf{1}\\\mathbf{1}&\mathbf{0}&\mathbf{8}&\mathbf{6}\end{bmatrix}$. **Show** that the matrices **A** and **B** are row equivalent to each other.

b) **Determine t**he value/s of α such that the following linear system:

$$\begin{array}{lllll} x & +2y & - & z & = 2\\ x & -2y & + & 3z & = 1\\ x & +2y & -(\alpha^2-3)\,z & & = \alpha\end{array}$$

has: **(i).** no solution; **(ii).** unique solution; **(iii).** infinitely many solutions.

Problem 3:

a) **Show** that any homogeneous system of linear equations either has only the trivial solution or infinitely many solutions and so every homogeneous linear system is consistent.

b) **Give** example of a homogeneous linear system with only the trivial solution.

c) **Give** example of a homogeneous linear system having infinitely many non-trivial solutions.

d) By using the Cramer's rule, **solve** the following system:

$$\begin{array}{llll} x & +2y & -\;\; z & = 2\\ x & +3y & +\;\; 3z & = 2\\ x & +3y & +\;\; 5z & = 4.\end{array}$$

Problem 4:

a) Let $\mathbf{W}=\{\mathbf{A}\in\mathbf{M}_{2\times 2}(\mathbb{R}):\mathbf{AB}=\mathbf{BA}\}$, where $\mathbf{B}=\begin{bmatrix}\mathbf{-1}&\mathbf{1}\\\mathbf{0}&\mathbf{1}\end{bmatrix}$. Then:

(i). Show that **W** is a vector subspace of the vector space $\mathbf{M}_{2\times 2}(\mathbb{R})$.

(ii). Find a basis and dimension of **W**.

b) Find a basis of the vector space $\mathbb{R}^3$ which contains the set $\{(1,1,0),(1,-1,0)\}$.

Problem 5:

a) **Show** that **A** ={ $\begin{bmatrix} \boldsymbol{\alpha} & \boldsymbol{\beta} \\ \gamma & \delta \end{bmatrix}$ ∈$\mathbf{M}_{2\times2}(\mathbb{R})$: **α+β = γ - δ**}is a vector subspace of $\mathbf{M}_{2\times2}(\mathbb{R})$. Also **find** a basis and dimension of the vector space **A**.

b) **Show** that **S**={**X**∈$\mathbf{M}_{2\times2}(\mathbb{R})$: **X** = **-** $\mathbf{X}^{\mathbf{T}}$} is a subspace of the vector space $\mathbf{M}_{2\times2}(\mathbb{R})$ and **show further** that the set {$\begin{bmatrix} \mathbf{0} & -\mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix}$} is a basis for **S**.

c) **Determine** whether {**X**∈$\mathbf{M}_{2\times2}(\mathbb{R})$: **X** = $\mathbf{X}^{\mathbf{T}}$} is a proper subspace of the vector space $\mathbf{M}_{2\times2}(\mathbb{R})$?

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