

Subject: 7.1

Date: / /

1, 2, 3, 4, 5

1) Find  $\|x\|_\infty$  and  $\|x\|_2$  for the following vectors:-

a.  $x = (3, -4, 0, \frac{3}{2})^t$ .

$$\|x\|_2 = \sqrt{9 + 16 + 0 + \frac{9}{4}} = \sqrt{27.25} \approx 5.2202.$$

$$\|x\|_\infty = \max\{|3|, |-4|, |0|, |\frac{3}{2}|\} = 4.$$

c.  $x = (\sin k, \cos k, 2^k)^t$  for a fixed positive integer  $k$ .

$$\|x\|_2 = \sqrt{\sin^2 k + \cos^2 k + 2^{2k}} = \sqrt{1 + 2^{2k}}$$

$$\|x\|_\infty = \max\{|\sin k|, |\cos k|, |2^k|\} = 2^k.$$

since  $|\sin k| \leq 1$ ,  $|\cos k| \leq 1$ .

d.  $x = (\frac{4}{(k+1)}, \frac{2}{k^2}, k^2 e^{-k})^t$  for a fixed positive integer  $k$ .

$$\|x\|_2 = \sqrt{\frac{16}{(k+1)^2} + \frac{4}{k^4} + k^4 e^{-2k}}$$

$$\|x\|_\infty = \max\left\{\frac{4}{k+1}, \frac{2}{k^2}, \frac{k^2}{e^k}\right\}$$

Subject: \_\_\_\_\_

Date: 7/ /

माना  $f(x) = x^2 + 2x + 1$  है।

$$f'(x) = 2x + 2$$

माना  $f(x) = x^2 + 2x + 1$  है।

$$f'(x) = 2x + 2$$



2) a. verify that the function  $\|\cdot\|_1$ , defined on  $\mathbb{R}^n$  by:

$$\|x\|_1 = \sum_{i=1}^n |x_i|,$$

is a norm on  $\mathbb{R}^n$ .

$$\textcircled{1} |x_i| \geq 0 \Rightarrow \|x\|_1 \geq 0.$$

$$\textcircled{2} \|x\|_1 = 0 \Leftrightarrow \sum |x_i| = 0 \Leftrightarrow |x_i| = 0, \forall i$$

$$\Leftrightarrow x = 0$$

$$\textcircled{3} \|\alpha x\|_1 = \sum |\alpha x_i| = |\alpha| \sum |x_i|$$

$$= |\alpha| \|x\|_1$$

$$\textcircled{4} \|x+y\|_1 = \sum |x_i + y_i| \leq \sum (|x_i| + |y_i|)$$

$$= \sum |x_i| + \sum |y_i|$$

$$= \|x\|_1 + \|y\|_1$$

b. Find  $\|x\|_1$  for the vectors given in Ex-1.  
(جواباً على المسألة)

c. prove that for all  $x \in \mathbb{R}^n$ ,  $\|x\|_1 \geq \|x\|_2$ .

$$x = (x_1, x_2, \dots, x_n).$$

$$\text{—} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$$

$$\|x\|_2 = \left( \sum |x_i|^2 \right)^{1/2}$$

$$\|x\|_2^2 = \sum |x_i|^2$$

Subject: \_\_\_\_\_

Date: / /

1. Let  $f(x) = x^2 + 2x + 1$  and  $g(x) = x^2 - 2x + 1$

Find  $(f+g)(x)$  and  $(f-g)(x)$

$(f+g)(x) = (x^2 + 2x + 1) + (x^2 - 2x + 1)$

$= x^2 + 2x + 1 + x^2 - 2x + 1$

$= 2x^2 + 2$

$(f-g)(x) = (x^2 + 2x + 1) - (x^2 - 2x + 1)$

$= x^2 + 2x + 1 - x^2 + 2x - 1$

$= 4x$

2. Let  $f(x) = 3x^2 - 2x + 1$  and  $g(x) = x^2 + 4x - 3$

Find  $(f+g)(x)$  and  $(f-g)(x)$

$(f+g)(x) = (3x^2 - 2x + 1) + (x^2 + 4x - 3)$

$= 3x^2 - 2x + 1 + x^2 + 4x - 3$

$= 4x^2 + 2x - 2$

$(f-g)(x) = (3x^2 - 2x + 1) - (x^2 + 4x - 3)$

$= 3x^2 - 2x + 1 - x^2 - 4x + 3$

$= 2x^2 - 6x + 4$

3. Let  $f(x) = 5x^2 - 3x + 2$  and  $g(x) = 2x^2 + 7x - 1$

Find  $(f+g)(x)$  and  $(f-g)(x)$

$(f+g)(x) = (5x^2 - 3x + 2) + (2x^2 + 7x - 1)$

$= 5x^2 - 3x + 2 + 2x^2 + 7x - 1$

$= 7x^2 + 4x + 1$

$(f-g)(x) = (5x^2 - 3x + 2) - (2x^2 + 7x - 1)$

$= 5x^2 - 3x + 2 - 2x^2 - 7x + 1$



Subject: \_\_\_\_\_

Date: / /

$$\|x\|_2^2 = \sum |x_i|^2$$

$$\leq \left(\sum |x_i|\right)^2$$

$$= \|x_1\|^2$$

$$\|x_2\|^2 \leq \|x_1\|^2$$

$$\Rightarrow \|x_2\| \leq \|x_1\|$$

we want to prove that:

$$\sum_{i=1}^n |x_i|^2 \leq \left(\sum_{i=1}^n |x_i|\right)^2$$

- i.e.  $\sum_{i=1}^n |x_i|^2 \leq \left(\sum_{i=1}^n |x_i|\right)^2$

① if  $n=2$

$$\sum_{i=1}^2 |x_i|^2 = |x_1|^2 + |x_2|^2$$

$$\begin{aligned} \left(\sum_{i=1}^2 |x_i|\right)^2 &= (|x_1| + |x_2|)^2 = |x_1|^2 + 2|x_1||x_2| + |x_2|^2 \\ &\geq \sum_{i=1}^2 |x_i|^2 \end{aligned}$$

② kisi kisi ke liye

i.e.  $\sum_{i=1}^k |x_i|^2 \leq \left(\sum_{i=1}^k |x_i|\right)^2$

Subject: \_\_\_\_\_

Date:    /    /



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③  $(k+1)$  terms

$$\sum_{i=1}^{k+1} |x_i|^2 \stackrel{?!!}{\leq} \left( \sum_{i=1}^{k+1} |x_i| \right)^2$$

$$\sum_{i=1}^{k+1} |x_i|^2 = |x_1|^2 + |x_2|^2 + \dots + |x_k|^2 + |x_{k+1}|^2$$

$$= \sum_{i=1}^k |x_i|^2 + |x_{k+1}|^2$$

$$\leq \left( \sum_{i=1}^k |x_i| \right)^2 + |x_{k+1}|^2$$

$$\leq \left( \sum_{i=1}^{k+1} |x_i| \right)^2$$

3. prove that the following sequences are convergent, and find their limits.

a.  $x^{(k)} = \left( \frac{1}{k}, e^{1-k}, \frac{-2}{k^2} \right)^t$ .

$$\lim_{k \rightarrow \infty} \frac{1}{k} = 0$$

$$\lim_{k \rightarrow \infty} e^{1-k} = \lim_{k \rightarrow \infty} e \cdot e^{-k} = \lim_{k \rightarrow \infty} \frac{e}{e^k} = 0$$

$$\lim_{k \rightarrow \infty} \frac{-2}{k^2} = 0$$

$$\therefore x^{(k)} \rightarrow (0, 0, 0).$$

Subject: \_\_\_\_\_

Date: / /





Subject: \_\_\_\_\_

Date: / /

$$b. x^{(k)} = (e^{-k} \cos k, k \sin(\frac{1}{k}), 3 + k^{-2})^t.$$

$$\lim_{k \rightarrow \infty} e^{-k} \cos k = \lim_{k \rightarrow \infty} \frac{\cos k}{e^k} = 0$$

$$-1 \leq \cos k \leq 1$$

$$\frac{-1}{e^k} \leq \frac{\cos k}{e^k} \leq \frac{1}{e^k}$$

$$\lim_{k \rightarrow \infty} k \sin(\frac{1}{k}) = \lim_{\frac{1}{k} \rightarrow 0} \frac{\sin(\frac{1}{k})}{\frac{1}{k}} = 1.$$

$$\lim_{k \rightarrow \infty} 3 + k^{-2} = \lim_{k \rightarrow \infty} (3 + \frac{1}{k^2}) = 3 + 0 = 3.$$

$$\therefore x^{(k)} \rightarrow (0, 1, 3).$$

$$c. x^{(k)} = (k e^{-k^2}, \frac{\cos k}{k}, \sqrt{k^2 + k} - k)^t.$$

$$\lim_{k \rightarrow \infty} k e^{-k^2} = \lim_{k \rightarrow \infty} \frac{k}{e^{k^2}} = \lim_{k \rightarrow \infty} \frac{1}{2ke^{k^2}} = 0$$

$$\lim_{k \rightarrow \infty} \frac{\cos k}{k} = 0$$

$$\lim_{k \rightarrow \infty} \frac{\sqrt{k^2 + k} - k \times \sqrt{k^2 + k} + k}{\sqrt{k^2 + k} + k}$$

$$= \lim_{k \rightarrow \infty} \frac{k^2 + k - k^2}{\sqrt{k^2 + k} + k} = \lim_{k \rightarrow \infty} \frac{k}{k(\frac{\sqrt{k^2 + k}}{\sqrt{k^2}} + 1)}$$

$$= \lim_{k \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{k}} + 1} = \frac{1}{2}$$

$$x^k \rightarrow (0, 0, \frac{1}{2}).$$

Subject: \_\_\_\_\_

Date: / /



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Subject:  $\|A\|_\infty = \max \sum |a_{ij}|$  (Thm 7.11)

4. Find  $\|A\|_\infty$  for the following matrices:

a.  $\begin{bmatrix} 10 & 15 \\ 0 & 1 \end{bmatrix}$

$$\|A\|_\infty = \max \{25, 1\} = 25$$

b.  $\begin{bmatrix} 10 & 0 \\ 15 & 1 \end{bmatrix}$

$$\|A\|_\infty = \max \{10, 16\} = 16$$

c.  $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

$$\|A\|_\infty = \max \{3, 4, 3\} = 4.$$

5. The following linear systems  $Ax=b$  have  $x$  as the actual solution and  $\tilde{x}$  as an approximate solution. Compute  $\|x - \tilde{x}\|_\infty$  and  $\|A\tilde{x} - b\|_\infty$ .

a.  $\frac{1}{2}x_1 + \frac{1}{3}x_2 = \frac{1}{63}$

$$\frac{1}{3}x_1 + \frac{1}{4}x_2 = \frac{1}{168}$$

$$x = \left(\frac{1}{7}, \frac{1}{6}\right)^t$$

$$\tilde{x} = (0.142, -0.166)^t$$

Subject: \_\_\_\_\_

Date: 11/11/11



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Subject:

Date:

$$\|x - \tilde{x}\|_{\infty} = \left\| \left( \frac{1}{7} - 0.142, \frac{1}{6} + 0.166 \right) \right\|_{\infty} \\ = 0.000857$$

$$\|A\tilde{x} - b\|_{\infty} = \left\| \begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 0.142 \\ -0.166 \end{bmatrix} - \begin{bmatrix} \frac{1}{63} \\ \frac{1}{168} \end{bmatrix} \right\|_{\infty} \\ = \left\| \begin{bmatrix} -0.000206 \\ -0.00012 \end{bmatrix} \right\|_{\infty} \\ = \max \{ 0.000206, 0.00012 \} \\ = \underline{0.000206}$$

Subject: \_\_\_\_\_

Date: / /



1, 2, 3, 4, 6

1. compute the eigenvalues and associated eigenvectors of the following matrices...

a.  $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)^2 - 1 = 0$$

$$4 - 4\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda_{1,2} = \frac{4 \pm \sqrt{4}}{2}$$
$$= \frac{4 \pm 2}{2}$$

$$\lambda_1 = 3 \quad \text{or} \quad \lambda_2 = 1$$

Subject: \_\_\_\_\_

Date: 27 / 1

27/1/2020  
The first part of the paper was very easy. I was able to complete it in 15 minutes. The second part was more difficult. I spent 30 minutes on it. The third part was the most difficult. I spent 45 minutes on it. The total time was 1 hour and 15 minutes.

1 - 5  
2 - 10  
3 - 15

DA 1/1/20  
1 - 5  
2 - 10  
3 - 15

1 - 5  
2 - 10  
3 - 15

1 - 5  
2 - 10  
3 - 15

1 - 5  
2 - 10  
3 - 15



① if  $\lambda = 1$ :

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$\therefore x_1 - x_2 = 0 \Rightarrow x_1 = x_2$$

$$\text{Let } x_2 = t \in \mathbb{R}.$$

$$\therefore x_1 = t.$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \end{bmatrix} ; t \in \mathbb{R}.$$

$\therefore$  eigen vector  $(1, 1)^t$ .

② if  $\lambda = 3$ :

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \xrightarrow{-R_1} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\therefore x_1 + x_2 = 0 \Rightarrow x_1 = -x_2$$

$$\text{Let } x_2 = t \in \mathbb{R}, x_1 = -t.$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -t \\ t \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \end{bmatrix} ; t \in \mathbb{R}.$$

$\therefore$  eigen vector  $(-1, 1)^t$ .

Subject: \_\_\_\_\_

Date: / /



2. Find the spectral radius for each matrix in Exercise 1...

$$\textcircled{a} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \lambda = 1 \text{ or } \lambda = 3.$$

$$\therefore \rho(A) = \max |\lambda| = \max \{1, 3\} = 3.$$

3. which of the matrices in Ex. 1 are convergent?

$$a. \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

"Thm. 7.17"

since  $\rho(A) = 3 > 1 \Rightarrow$  not convergent.

4. Let  $A_1 = \begin{bmatrix} 1 & 0 \\ 1/4 & 1/2 \end{bmatrix}$  and  $A_2 = \begin{bmatrix} 1/2 & 0 \\ 16 & 1/2 \end{bmatrix}$ . Show that  $A_1$  is not convergent, but  $A_2$  is convergent.

$$\begin{vmatrix} 1-\lambda & 0 \\ 1/4 & 1/2-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)(1/2-\lambda) = 0$$

$$\Rightarrow \frac{1}{2} = \lambda - \frac{1}{2}\lambda + \lambda^2 = 0$$

$$\Rightarrow \lambda^2 - \frac{3}{2}\lambda + \frac{1}{2} = 0$$

$$\Rightarrow \lambda = 1 \text{ or } \lambda = \frac{1}{2}$$

$$\rho(A_1) = \max \{1, \frac{1}{2}\} = 1 \not< 1 \Rightarrow \text{not convergent.}$$

Subject: \_\_\_\_\_

Date: / /

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Subject: \_\_\_\_\_

Date: / /

$$\begin{vmatrix} \frac{1}{2} - \lambda & 0 \\ 16 & \frac{1}{2} - \lambda \end{vmatrix} = 0 \Rightarrow \left(\frac{1}{2} - \lambda\right)^2 = 0$$
$$\Rightarrow \lambda = \frac{1}{2}, \frac{1}{2}$$

$$\therefore \rho(A_2) = \max\left\{\frac{1}{2}, \frac{1}{2}\right\} = \frac{1}{2} < 1.$$

$\therefore A_2$  is convergent.

6. Show that if  $\lambda$  is an eigen value of a matrix  $A$  and  $\|\cdot\|$  is a vector norm, then an eigenvector  $x$  associated with  $\lambda$  exists with  $\|x\| = 1$ .

— Let  $x$  be the eigenvector associated with  $\lambda$ , with  $\|x\| = a$ .

then  $a \neq 0$ , since  $x \neq 0$ .

Now, consider the vector  $\frac{x}{a}$ , then  $\frac{x}{a} \neq 0$

$$\text{and } (A - \lambda I)\left(\frac{x}{a}\right) = \frac{1}{a} [(A - \lambda I)x]$$
$$= \frac{1}{a} [0] = 0$$

hence,  $\frac{x}{a}$  is an eigen vector, and:

$$\left\|\frac{x}{a}\right\| = \frac{1}{a} \|x\| = \frac{1}{\|x\|} \|x\| = 1.$$

Subject: \_\_\_\_\_

Date: / /



Subject:

7.4

1-2-3

Date:

1) Compute the condition numbers of the following matrices relative to  $\|\cdot\|_\infty$ .

(a) 
$$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} \end{bmatrix}$$

$$\|A\|_\infty = \max\left\{\frac{1}{2} + \frac{1}{3}, \frac{1}{3} + \frac{1}{4}\right\} = \max\left\{\frac{5}{6}, \frac{7}{12}\right\} = \frac{5}{6}$$

$$A^{-1} = \frac{1}{\frac{1}{8} - \frac{1}{9}} \begin{bmatrix} \frac{1}{4} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{2} \end{bmatrix} = \frac{1}{\frac{1}{72}} \begin{bmatrix} \frac{1}{4} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{2} \end{bmatrix} = 72 \begin{bmatrix} \frac{1}{4} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 18 & -24 \\ -24 & 36 \end{bmatrix}$$

$$\|A^{-1}\|_\infty = \max\{42, 60\} = 60$$

$$\begin{aligned} \therefore K(A) &= \|A\|_\infty \cdot \|A^{-1}\|_\infty \\ &= \frac{5}{6} (60) \\ &= 50 \end{aligned}$$



$$\begin{bmatrix} 1 & & & \\ 0 & -1 & & \\ 0 & 1 & -1 & \\ & 0 & -1 & \\ & & & -1 \end{bmatrix}$$

Date: / /

$$\|A\|_\infty = \max\{3, 2, 1\} = 3.$$

$$\begin{bmatrix} 1 & & & & & \\ 0 & -1 & & & & \\ 0 & 1 & -1 & & & \\ & & & -1 & & \\ & & & & 0 & 0 \\ & & & & 0 & 1 \end{bmatrix} \xrightarrow{\substack{-1R_3 \\ 1R_2+R_1}} \begin{bmatrix} 1 & 0 & -2 & & & \\ 0 & 1 & & & & \\ 0 & 0 & 1 & & & \\ & & & -1 & & \\ & & & & 0 & 0 \\ & & & & 0 & -1 \end{bmatrix}$$

$R_3+R_2$

$\downarrow$

$$\begin{bmatrix} 1 & & & & & \\ 0 & 1 & & & & \\ 0 & 0 & -2 & & & \\ & & & -1 & & \\ & & & & 0 & 0 \\ & & & & 0 & 1 \end{bmatrix} \xrightarrow{\substack{-1R_3 \\ 1R_2+R_1}} \begin{bmatrix} 1 & 0 & -2 & & & \\ 0 & 1 & & & & \\ 0 & 0 & 1 & & & \\ & & & -1 & & \\ & & & & 0 & 0 \\ & & & & 0 & -1 \end{bmatrix}$$

$A^{-1}$

$$\begin{bmatrix} 1 & & & \\ 1 & -2 & & \\ & -1 & & \\ 0 & & 1 & \end{bmatrix}$$

$$\max\{4, 2, 1\} = 4.$$

$$\|A\|_\infty \cdot \|A^{-1}\|_\infty = 3 \times 4 = 12.$$



Subject:

Date:

2) The following linear systems  $Ax=b$  have  $x$  as the actual solution and  $\tilde{x}$  as an approximation solution. Using the results of Ex. 1, compute  $\|x - \tilde{x}\|_\infty$  and  $K_\infty(A) = \frac{\|b - A\tilde{x}\|_\infty}{\|A\tilde{x}\|_\infty}$

$$\textcircled{a} \begin{cases} \frac{1}{2}x_1 + \frac{1}{3}x_2 = \frac{1}{63} \\ \frac{1}{3}x_1 + \frac{1}{4}x_2 = \frac{1}{168} \end{cases}$$

$$x = \left(\frac{1}{7}, \frac{1}{6}\right)^t$$

$$\tilde{x} = (0.142, -0.166)^t$$

$$\|A\|_\infty = \frac{5}{6}$$

$$K_\infty(A) = 50$$

$$\|x - \tilde{x}\|_\infty = (0.000857, -0.000666)^t$$

$$\Rightarrow \|x - \tilde{x}\|_\infty = 0.000857$$

$$b - A\tilde{x} = \begin{bmatrix} \frac{1}{63} \\ \frac{1}{168} \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 0.142 \\ -0.166 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{63} \\ \frac{1}{168} \end{bmatrix} - \begin{bmatrix} 0.01566666666 \\ 0.00583333333 \end{bmatrix}$$

Subject:

Date: / /

1. Find the inverse of the matrix  

$$A = \begin{bmatrix} 2 & -2 \\ -1.0001 & 1 \end{bmatrix}$$

Solution:  

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\det(A) = (2)(1) - (-2)(-1.0001) = 2 - 2.0002 = -0.0002$$

$$A^{-1} = \frac{1}{-0.0002} \begin{bmatrix} 1 & 2 \\ 1.0001 & -2 \end{bmatrix}$$

$$A^{-1} = -5000 \begin{bmatrix} 1 & 2 \\ 1.0001 & -2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -5000 & -10000 \\ -5000.5 & 10000 \end{bmatrix}$$

$$\frac{-1}{0.0002} \begin{bmatrix} 2 & -2 \\ -1.0001 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -10000 & +10000 \\ 5000.5 & -5000 \end{bmatrix}$$



Subject:

Date:

$$b - A\tilde{x} = \begin{bmatrix} 0.000206349203 \\ 0.0001190476524 \end{bmatrix}$$

$$\|b - A\tilde{x}\|_{\infty} = 0.000206$$

$$\begin{aligned} \therefore K_{\infty}(A) &= \frac{\|b - A\tilde{x}\|_{\infty}}{\|A\|_{\infty}} = \frac{(50)(0.000206)}{\left(\frac{5}{6}\right)} \\ &= \frac{10}{(50)(0.000206)(6)} \\ &= 0.01236 \end{aligned}$$

3) The linear system:

$$\begin{bmatrix} 1 & 2 \\ 1.0001 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.0001 \end{bmatrix}$$

has solution  $(1, 1)^t$ . Change  $A$  slightly to

$$\begin{bmatrix} 1 & 2 \\ 0.9999 & 2 \end{bmatrix}$$

and consider the linear system:

$$\begin{bmatrix} 1 & 2 \\ 0.9999 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.0001 \end{bmatrix}$$

Subject: \_\_\_\_\_

Date: / /

(7.24):

$$\frac{\|x - \tilde{x}\|}{\|x\|} \leq \frac{\kappa(A) \|A\|}{\|A\| - \kappa(A) \|S(A)\|} \left( \frac{\|S(b)\|}{\|b\|} + \frac{\|S(A)\|}{\|A\|} \right)$$

$$\frac{\|x - \tilde{x}\|_{\infty}}{\|x\|_{\infty}} = \frac{\max\{2, 1\}}{\max\{1, 1\}} = \frac{2}{1} = 2$$

$$S(A) = \begin{bmatrix} 0 & 0 \\ -0.0002 & 0 \end{bmatrix}, S(b) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\|S(A)\|_{\infty} = 0.0002, \|S(b)\|_{\infty} = 0$$

$$\begin{aligned} & \frac{\kappa(A) \|A\|}{\|A\| - \kappa(A) \|S(A)\|} \left( \frac{\|S(b)\|}{\|b\|} + \frac{\|S(A)\|}{\|A\|} \right) \\ &= \frac{(60000)(3)}{3 - (60000)(0.0002)} \left( \frac{0}{3.0001} + \frac{0.0002}{3} \right) \\ &= \frac{(60000)(3)}{-93} \left( \frac{1}{15000} \right) \\ &= \frac{-20000}{15000} = -\frac{4}{3} \end{aligned}$$



Subject: \_\_\_\_\_

Date: / /

$$2 < \frac{-4}{3}$$

(-X-)

الملاحظة (7.24) ليست صحيحة

Subject: \_\_\_\_\_

Date: / /

1-2-3

(2)

10/10/10