PHYS 551-505
HANDOUT 8 – Quantum scattering theory.

1. Show that in the case where the scattering is at $\theta = 90^0$ we have:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{indistinguishable}} = 4\left|f\left(\pi/2\right)\right|^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{distinguishable}} = 2\left|f\left(\pi/2\right)\right|^2.$$

2. Express the differential cross-section for two identical particles scattering in terms of the scattering amplitude, assuming that the interaction potential has spherical symmetry. Treat the case of bosons and fermions. What happens if the beam of fermions is not polarized?

3. Using the answer from question 2, express the differential cross-section for identical particles scattering in terms of the scattering amplitude, assuming that the beams of scattered particles are randomly polarized.

4. (a) Verify that, outside the range of a short-range potential, the wave function

$$\psi\left(r,\theta\right) = \frac{1}{r}\left(1 + \frac{i}{kr}\right)\exp\left(ikr\right)\cos\theta$$

represents a p-wave.
(b) A beam of particles represented by the plane wave exp(ikz) is scattered by an impenetrable sphere of radius a, where $ka \ll 1$. By considering only s and p components in the scattered wave, show that, to order $\left(ka\right)^2$, the differential cross-section for scattering at an angle θ is

$$\frac{d\sigma}{d\Omega} = a^2\left\{1 - \frac{1}{3}\left(ka\right)^2 + 2\left(ka\right)^2\cos\theta\right\}$$

(the value of $\cos^2\theta$ averaged over all directions is $1/3$)

5. Particles of a given energy scatter on an infinitely hard sphere of radius a.

(a) Calculate the *phase shift* δ_l.
(b) For s-waves ($l = 0$), find the values of the energy for which the partial cross section becomes maximal.
(c) Consider the case of low energies ($ka \ll 1$), write an approximate expression for δ_l and explain why the cross section

is dominated by *s*-waves and is isotropic. Compare the low-energy cross section with the *geometric value* πa^2.

6. Consider the scattering of a particle from a real spherically symmetric potential. If $d\sigma(\theta)/d\Omega$ is the differential cross section and σ is the total cross section, show that

$$\sigma \le \frac{4\pi}{k}\sqrt{\frac{d\sigma(0)}{d\Omega}}.$$

Verify this inequality explicitly for a general central potential using the partial-wave expansion of the scattering amplitude and the cross-section.

7. A particle of mass *m* is scattered from a spherical repelling potential of radius *R*:

$$V(r) = \begin{cases} V_0 & r \le R \\ 0 & r \ge 0 \end{cases}.$$

Find the total cross-section which corresponds to the *s*-wave contribution.

8. Neutrons of mass *m* and energy *E* are incident on a spherically symmetric, square-well, attractive potential of depth *W* and range *a*, representing the nuclear force between the neutron and a nucleus. If the velocity is $v << \hbar / ma$, show that : a) the scattering is spherically symmetric, b) the *s*-wave phase shift δ satisfies $j\tan(ka+\delta) = k\tan ja$ where

$$k^2 = \frac{2mE}{\hbar^2}, \quad j^2 = \frac{2m(W+E)}{\hbar^2}.$$

9. **9.** Calculate the differential cross section in the first Born approximation for the scattering of an unpolarized beam of electrons interacting through a Yukawa type potential $V(r) = -g\frac{e^{-\lambda r}}{r}$ where the scattering amplitude is given by $f_B = \frac{2mg}{\hbar^2(\lambda^2+q^2)}$.

10. Consider the case of scattering on a hard sphere of radius *r=a* calculate partial scattering cross-section for an s-wave. Calculate the phase shift δ_1.[1]

You may use for calculations of indefinite integrals the Wolfram On-Line Integrator at: http://integrals.wolfram.com/index.jsp.

[1] http://fac.ksu.edu.sa/vlempesis