PHYS 505
HANDOUT 7 – On the Born approximation in quantum scattering theory.

1. Show that in the scattering by a Yukawa potential given by $V(r) = -g\frac{e^{-\lambda r}}{r}$ the scattering amplitude is given by $f_B = \frac{2mg}{\hbar^2\left(\lambda^2 + q^2\right)}$.

2. Find the differential cross section area, in the Born approximation, for the Gaussian potential that has the form: $V(r) = V_0 e^{-a^2 r^2}$.

3. Compare the differential cross-section of a Gaussian potential $V_G(r) = \frac{V_0}{\sqrt{4\pi}} e^{-r^2/4a^2}$ with that for Yukawa potential $V_Y(r) = \frac{V_0 a}{r} e^{-r/a}$. Make a plot for both quantities against *qa*. Also check the case $qa << 1$.

4. Find in the Born approximation the differential and total cross-section for scattering in the field $V(r) = V_0 e^{-r/a}$.

5. Using the Born approximation express the differential cross-section for the scattering of an electron from a spherical symmetric charge distribution $\rho(r)$ in the following two cases:
   (a) A uniform charge distribution
   $$\rho(r) = \begin{cases} 3q/4\pi R^3 & r \le R \\ 0 & r > R \end{cases}$$
   (b) A Gaussian charge distribution
   $$\rho(r) = \begin{cases} qe^{-r^2/R^2}/\pi^{3/2}R^3 & r \le R \\ 0 & r > R \end{cases}$$

6. For the scattering in the spherically symmetric potential $V(\mathbf{r}) = g\delta(\mathbf{r})$, (where $g > 0$ a positive constant and $\delta(\mathbf{r})$ the 3-dimensional δ function, we have $f_B(\theta) = -\frac{mg}{2\pi\hbar^2}$.

7. A particle of mass *m* is scattered from a spherical repelling potential of radius *R*:
   $$V(r) = \begin{cases} V_0, & r \le R \\ 0, & r > R \end{cases}.$$
   Calculate the total cross-section using the Born approximation in the limit of low energies ($qR \to 0$).

8. Show that if the scattering potential has a translation invariance property $V(\mathbf{r}+\mathbf{R}) = V(\mathbf{r})$ where $\mathbf{R}$ is a constant vector, then the Born approximation scattering vanishes unless $\mathbf{q}\cdot\mathbf{R} = 2\pi n$ with $n = 0, 1, 2, \ldots$

You may use for calculations of indefinite integrals the Wolfram On-Line Integrator at: http://integrals.wolfram.com/index.jsp.