PHYS 454
HANDOUT 7 – *Continuous spectrum in square wells*

Tunneling Through a Potential Energy Barrier

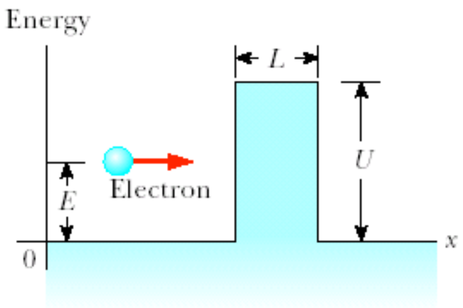
1. Given that for a particle with energy higher than the height of the potential barrier

$$A = \frac{i\left(k'^2 - k^2\right)\sin\left(k'L\right)}{2k'k\cos\left(k'L\right) - i\left(k'^2 + k^2\right)\sin\left(k'L\right)}$$

Derive the expressions for *R* and *T.*

2. Derive the expressions for *R* and *T* for a particle with lower energy than the height of the potential barrier.

3. An electron with kinetic energy $E = 5.00$ eV is incident on a barrier with thickness $L = 0.200$ nm and height $V_0 = 10.0$ eV. What is the probability that the electron (a) will tunnel through the barrier? (b) will be reflected?



Figure

4. If outside of an electron surface there exists a uniform electric field $\tilde{E}$, the potential energy of the electrons inside and outside of the metal is given by:

$$V(x) = \begin{cases} -V_0 & x < 0 \quad \text{(inside the metal)} \\ -e\tilde{E}x & x \geq 0 \quad \text{(outside the metal)} \end{cases}$$

Where the field inside the metal has been assumed uniform for simplicity. Show that for an electron of energy E=-W the escape probability , due to the external field is given by

$$T(W) \approx \exp\left(-\frac{4\sqrt{2m}}{3\hbar e\tilde{E}} W^{3/2} \right).$$

This effect is known as cold electron emission in contrast to the thermionic emission that is caused when we heat a metal.

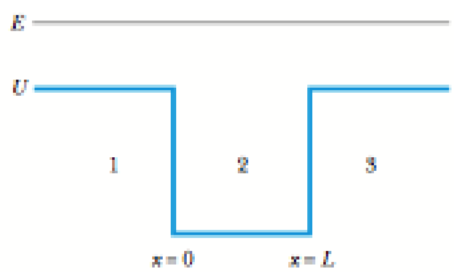
5. Two copper conducting wires are separated by an insulating oxide layer (CuO). Modeling the oxide layer as a square barrier of height 10.0 eV estimate the transmission coefficient for penetration by 7.00 eV if the layer thickness is (a) 5.00 nm and (b) 1.00 nm.

6. A 1.00 mA current of electrons in one of the wires of the problem 5 is incident on the oxide layer. How much of this current passes through the layer to the adjacent wire if the electron energy is 7.00 eV and the layer thickness is 1.00 nm? What becomes of the remaining current?

7. What happens to the transmission coefficient in an orthogonal barrier $E >> V_0$? What if E approaches V_0?

8. Estimate the leakage current due to tunneling that passes across a parallel-plate capacitor charged to a potential difference of 10 kV. Take the plate separation to be d=0.010 mm and the plate area to be $A = 1.00\ \text{cm}^2$.

9. Consider the scattering of particles from the potential well shown below. Show that waves reflected from the well edges at $x=0$ and $x=L$ will cancel completely if $2L = \lambda_2$, where λ_2 is the de Broglie wavelength of the particle in region 2. [This is a crude model for the Ramsauer-Townsend effect observed in the collisions of slow electrons with noble gas atoms like argon, krypton, and xenon. Electrons with just the right energy are diffracted around these atoms as if there were no obstacle in their path (perfect transmission). The effect is peculiar to nobble gases because their closed-shell configurations produce atoms with abrupt outer boundaries]



10. An electron and a proton move towards an orthogonal potential barrier with the same energy. Which particle has the greater probability to go through?

11. Using the tunneling model, estimate the half-lives for a decay of the radioactive elements Thorium and Polonium. The energy of the ejected alpha particles is 4.05 MeV and 8.95 MeV, respectively, and the nuclear size is about 9.00 fm in both cases.

12. A particle with energy E is scattered by an orthogonal potential barrier V_0 such that $E > V_0$. What is the minimum transmission coefficient and when this can be achieved?