PHYS 551-505

HANDOUT 3 – On spin

1. Show that spin cannot correspond to a rotation of an electron around an axis passing through its center of mass.

2. Calculate the average values of the spin components when its state is described by the vector
$$X = \frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix}.$$

3. The average value of the z component of the spin of a particle with $s = 1/2$ is $-\hbar/6$. What are the probabilities to find the particle with its spin "up" or "down" along z axis.

4. Show that when a particle is at a state with a certain projection of spin along z-axis – let's say spin "up" – the average values of the two other components (along x and y) are equal to zero. What happens with the corresponding uncertainties Δs_x, Δs_y?

5. A particle with spin $s = 1/2$ is in a spin "up" state along z. Calculate the probabilities to find it with spin "up" or spin "down" along an axis in the direction of unit vector **n** which makes an angle θ with the z-axis.

6. Construct the spin states with a certain projection along x axis $s_x = \pm\hbar/2$. Repeat the same problem along y.

7. The state of a particle with spin $s = 1/2$ is described by the vector
$$X = \frac{1}{3}\begin{pmatrix}1+2i\\2\end{pmatrix}.$$
What are the probabilities to find the particle with spin +1/2 or -1/2 along the x axis?

8. Construct the spin matrices for particles with $s = 1$.

9. A particle with spin $s = 1$ is at a state with a definite projection $s_x = +\hbar$ along x axis. Calculate the probabilities to find the particle with spin "up" $(s_z = +\hbar)$, spin "down" $(s_z = -\hbar)$ and spin "horizontal" $(s_z = 0)$. Also calculate the corresponding uncertainty Δs_z.

10. For the generic spin state of a particle with $s = 1$

$$X = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

write the general expressions for:

(a) the probabilities to find the particle with $s_z = 0$, $s_z = +\hbar$, $s_z = -\hbar$.

(b) the probabilities to find the particle with $s_x = 0$, $s_x = +\hbar$, $s_x = -\hbar$.

(c) Show the above results in the specific case where

$$X \approx \begin{pmatrix} 2+i \\ \sqrt{2} \\ 1+i \end{pmatrix}.$$

11. Construct the spin matrices for particles with $s = 3/2$.

12. The state of a particle with spin s = 1/2 is described by the vector

$$X = \frac{1}{\sqrt{6}} \begin{pmatrix} 1+i \\ 2 \end{pmatrix}.$$

What are the probabilities to find the particle with spin +1/2 or -1/2 along the z and along the x axis?

13. The state of a particle with spin s = 1/2 is described by the vector

$$X = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}.$$

(a) Determine the constant A.

(b) Find the expectation values $\langle s_x \rangle$, $\langle s_y \rangle$, $\langle s_z \rangle$.

(c) Find the "uncertainties" Δs_x, Δs_y, Δs_z.

14. Find the matrices $\mathbf{S}^2$, $\mathbf{S}_z$, $\mathbf{S}_x$, $\mathbf{S}_y$ (in the base of the common eigenvectors of $\mathbf{S}_z$, $\mathbf{S}^2$).

15. Find the eigenvalues and eigenvectors of the operator $\mathbf{S}_x$.

16. The state of a spin of a particle (s=1/2) is:

$$\left|\psi(t)\right\rangle = \frac{1}{\sqrt{2}}e^{i\omega t}\left|+\right\rangle + \frac{1}{\sqrt{2}}e^{-i\omega t}\left|-\right\rangle,$$

where $\left|\pm\right\rangle$ the common eigen-states of $\mathbf{S}^2$, $\mathbf{S}_z$. A) What is the probability a time instant t to measure $S_y = \pm\hbar/2$; B) What is the average value of $\left\langle S_y \right\rangle$?

17. a) For a particle with spin $1/2$ write the projection of the spin $S_n = \hat{S}\hat{\mathbf{n}}$ on an axis n (where $\hat{\mathbf{n}}$ the unit vector) in a matrix form in Cartesian coordinates. b) Show that the projection has eigenvalues $\pm\hbar/2$. c) Express the operator in spherical coordinates. d) Find the eigenvectors of this operator.