بسم الله الرحمن الرحيم

# ENGINEERING MANAGEMENT

# (GE 404)



## LECTURE #12

## Time-Cost Trade-Offs

# Contents

# Objectives of the Present lecture



- *To discuss project time-cost relationship*
- *To explain the steps involved in project crashing*

# Time-Cost Trade-offs (Time-Cost Relationship)



- There is a relationship between a **project's time to completion and its cost**.
- By understanding the time-cost relationship, one is better able to predict the **impact of a schedule change on project cost**.
- Time-cost trade-off, in fact, is an **important management tool** for **overcoming one of the critical path method limitations of being unable to bring the project schedule to a specified duration**.

# Reasons to Reduce Project Durations



- To avoid late penalties
- To realize incentives for timely or early competition of a project
- To beat the competition to the market (influences Bid price)
- To free resources for use on other projects
- To reduce the indirect costs
- To complete a project when weather conditions make it less expensive

# Methods to reduce Durations



1. Overtime: Have the existing crew work overtime. This **increase the labor costs** due to increase pay rate and decrease productivity.
2. Hiring and/or Subcontracting:
   a) Bring in additional workers to enlarge crew size. This **increases labor costs** due to overcrowding and poor learning curve.
   b) Add subcontracted labor to the activity. This **almost always increases the cost** of an activity unless the subcontracted labor is far more efficient.
3. Use of advanced technology: Use better/more advanced equipment. This will **usually increase costs** due to rental and transport fees. If **labor costs (per unit) are reduced, this could reduce costs**.

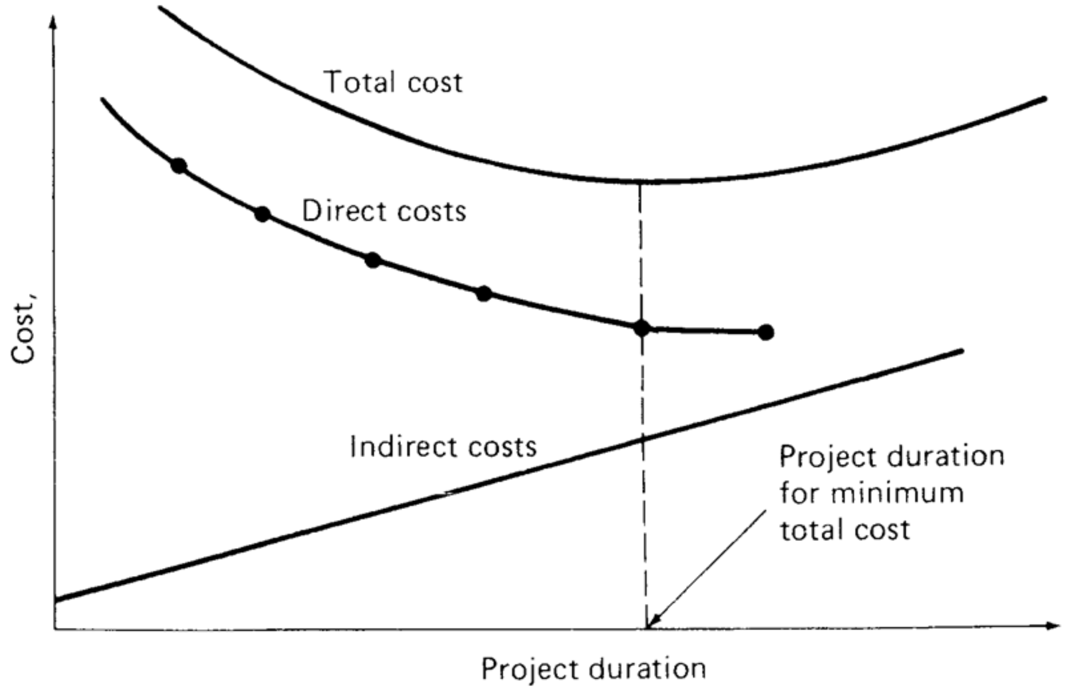
# Types of Costs



- **Direct Costs**
  - Direct costs are those **directly associated with project activities** such as cost of **labor**, **equipment** and **materials**, **salaries**.
  - If the **duration of activities is decreased** in order to decrease project completion time, the **direct cost generally increase** since more resources **must be allocated to accelerate** the activity.
- **Indirect Costs**
  - Indirect costs are those **overhead costs** that are not directly associated with specific project activities such as **office space**, **administrative staff**, and **taxes**.
  - Such costs tend to be relatively steady per unit time over the life of the project.
  - Total **indirect costs increase as the project duration increases**.
- **Note**: Project cost is the **sum** of the **direct** and **indirect costs**.

# Project Time-Cost Relationship





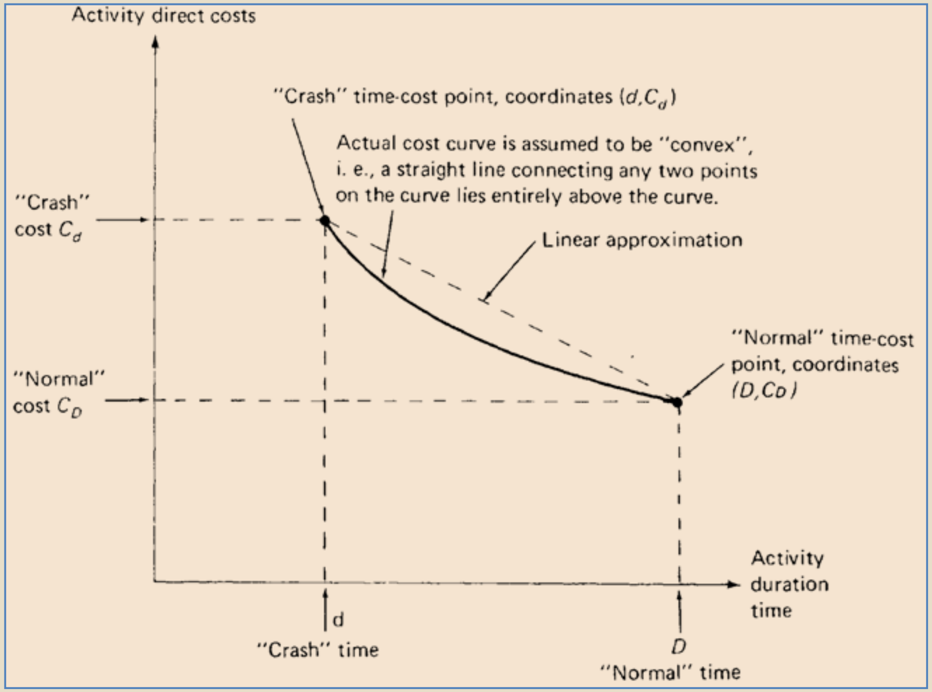

Determining project schedule for minimum total cost.

Note: It should **never be assumed** that the **quantity of resources** deployed and the **task duration** are **inversely related**.
Thus one should never automatically assume that the work that can be done by one man in 16 weeks can actually be done by 16 men in one week.

# Project Crashing and Cost Slope



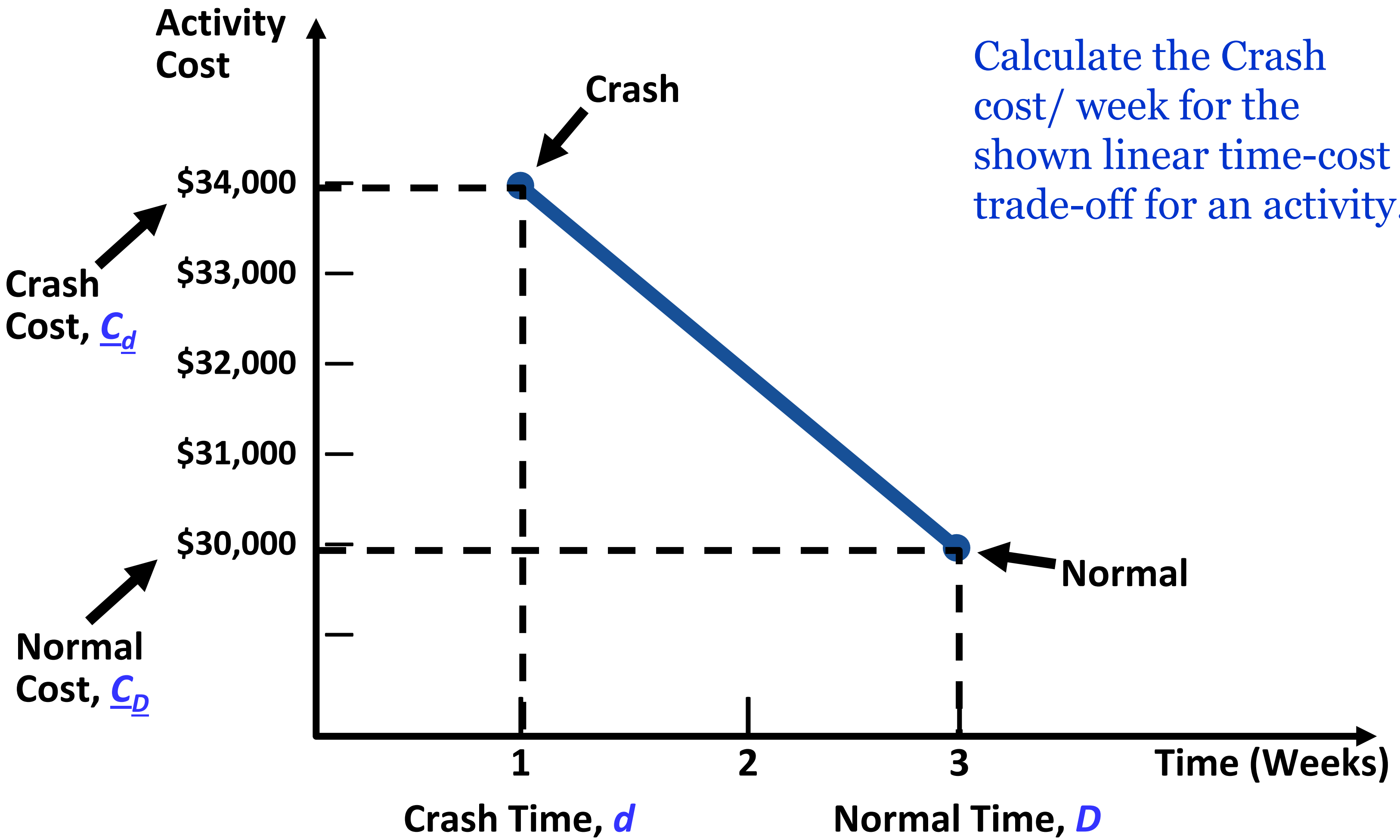
- **Shortening** the duration of a project is called **project crashing**
- The **minimum possible duration** of an activity (which implies maximum cost) is called the **Crashed duration** and corresponding cost is called **Crashed cost**
- Thus the **crash time is the *shortest* time** in which an activity can be completed
- Duration of an activity which implies **minimum direct cost** is called the **normal duration** and corresponding cost is called **normal cost**
- The slope of the line connecting the normal point (lower point) and the crash point (upper point) is called the **cost slope** of the activity.
- The slope of this line can be calculated mathematically by knowing the coordinates of the normal and crash points.




$$\text{Cost slope} = \frac{(\text{Crash cost - Normal cost})}{(\text{Normal duration - Crash duration})} = \frac{(C_d - C_D)}{(D - d)}$$

# Problem-1






Calculate the Crash cost/ week for the shown linear time-cost trade-off for an activity.

# Solution



$$\text{Cost slope} = \frac{(\text{Crash cost - Normal cost})}{(\text{Normal duration - Crash duration})} = \frac{(C_d - C_D)}{(D - d)}$$

$$= \frac{(\$34{,}000 - \$30{,}000)}{(3-1)} = \$2{,}000 \,/\, \text{Week}$$

$$\Rightarrow \text{Crash cost/week} = \text{Cost slope} = \$2{,}000 \,/\, \text{Week} \qquad \textit{Ans.}$$

# Compressing or Crashing the Project-Schedule



- Compressing or **Crashing** the project schedule refers to the **acceleration of the project activities** in order to complete the project sooner.
- The **time required** to complete a project is **determined by** the **critical path**, so **to compress** a project schedule one must focus **on critical path activities**.

# Basic Steps in Project Crashing



1. Compute the crash cost per time period
2. Using current activity times, find the critical path and identify the critical activities
3. If there is only one critical path, then select the activity on this critical path that (a) can still be crashed, and (b) has the smallest crash cost per period.
4. If there is **more than one critical path**, then **select one activity from each critical path** such that (a) each selected activity can still be crashed, and (b) the **total crash cost of all selected activities is the smallest**.
5. Note that the same activity may be common to more than one critical path.
6. **Update all activity times**. If the desired due date has been reached, stop. If not, return to Step 2.

# Problem-2

For the small project shown in the table, it is required reduce the project duration by

(i) 2 periods.

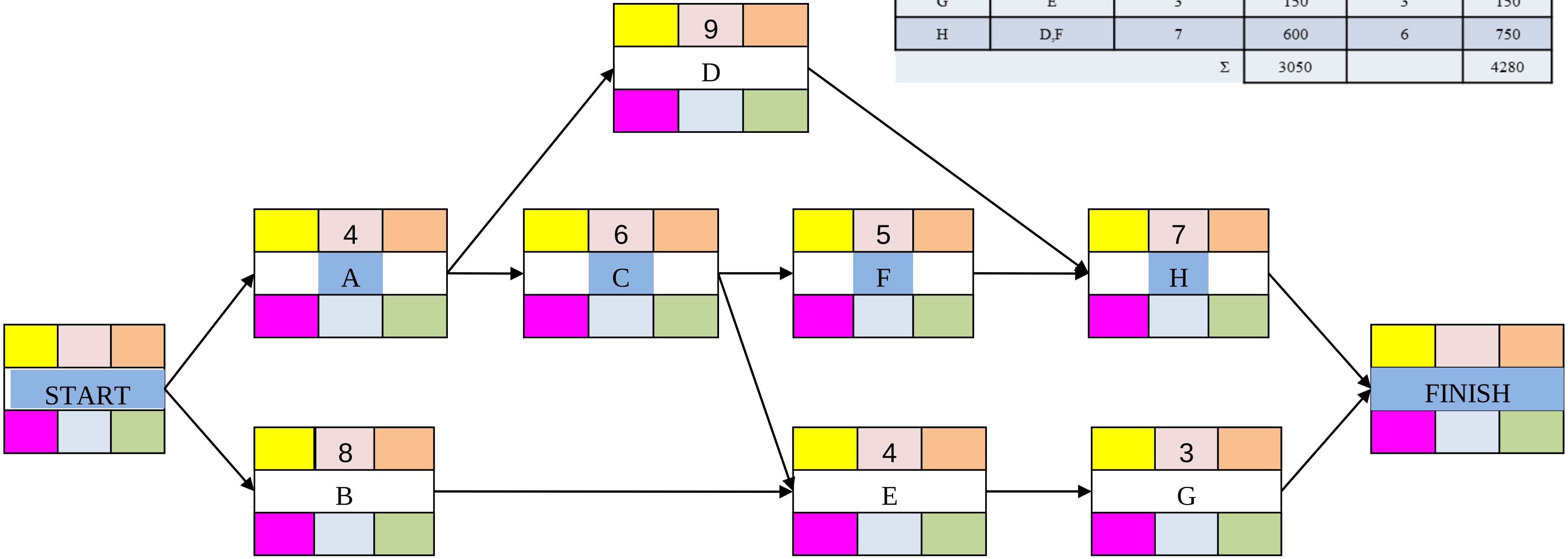
(ii) 5 periods.

| Activity | Precedence | Normal | | Crash | |
|---|---|---|---|---|---|
| | | Time, day | Cost, $ | Time, day | Cost, $ |
| A | - | 4 | 210 | 3 | 280 |
| B | - | 8 | 400 | 6 | 560 |
| C | A | 6 | 500 | 4 | 600 |
| D | A | 9 | 540 | 7 | 600 |
| E | B,C | 4 | 500 | 1 | 1100 |
| F | C | 5 | 150 | 4 | 240 |
| G | E | 3 | 150 | 3 | 150 |
| H | D,F | 7 | 600 | 6 | 750 |
| | | Σ | 3050 | | 4280 |

# Solution
# Step 1: Develop Network

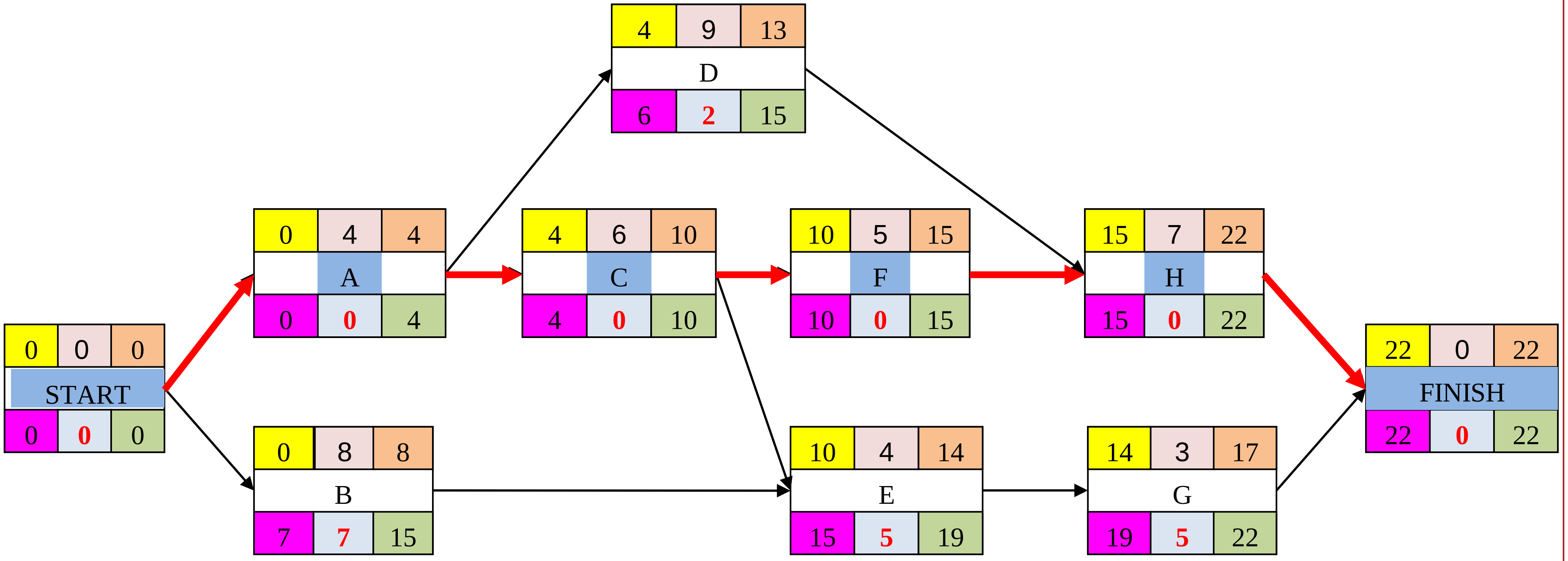


| Activity | Precedence | Normal | | Crash | |
|---|---|---|---|---|---|
| | | Time, day | Cost, $ | Time, day | Cost, $ |
| A | - | 4 | 210 | 3 | 280 |
| B | - | 8 | 400 | 6 | 560 |
| C | A | 6 | 500 | 4 | 600 |
| D | A | 9 | 540 | 7 | 600 |
| E | B,C | 4 | 500 | 1 | 1100 |
| F | C | 5 | 150 | 4 | 240 |
| G | E | 3 | 150 | 3 | 150 |
| H | D,F | 7 | 600 | 6 | 750 |
| | | Σ | 3050 | | 4280 |



# Step 2:Calculate Times and Find CP






- Project completion time = 22 working days
- Critical Path: A, C, F, H.

| Activity | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| Total float | 0 | 7 | 0 | 2 | 5 | 0 | 5 | 0 |
| Free float | 0 | 2 | 0 | 2 | 0 | 0 | 5 | 0 |

# Step 3: Calculate Cost Slope



| Activity | Precedence | Normal | | Crash | | Cost Slope, $/day |
|---|---|---|---|---|---|---|
| | | Time, day | Cost, $ | Time, day | Cost, $ | |
| A | - | 4 | 210 | 3 | 280 | 70 |
| B | - | 8 | 400 | 6 | 560 | 80 |
| C | A | 6 | 500 | 4 | 600 | 50 |
| D | A | 9 | 540 | 7 | 600 | 30 |
| E | B,C | 4 | 500 | 1 | 1100 | 200 |
| F | C | 5 | 150 | 4 | 240 | 90 |
| G | E | 3 | 150 | 3 | 150 | ** |
| H | D,F | 7 | 600 | 6 | 750 | 150 |
| | | Σ | 3050 | | 4280 | |

Note:

1- G can not expedite

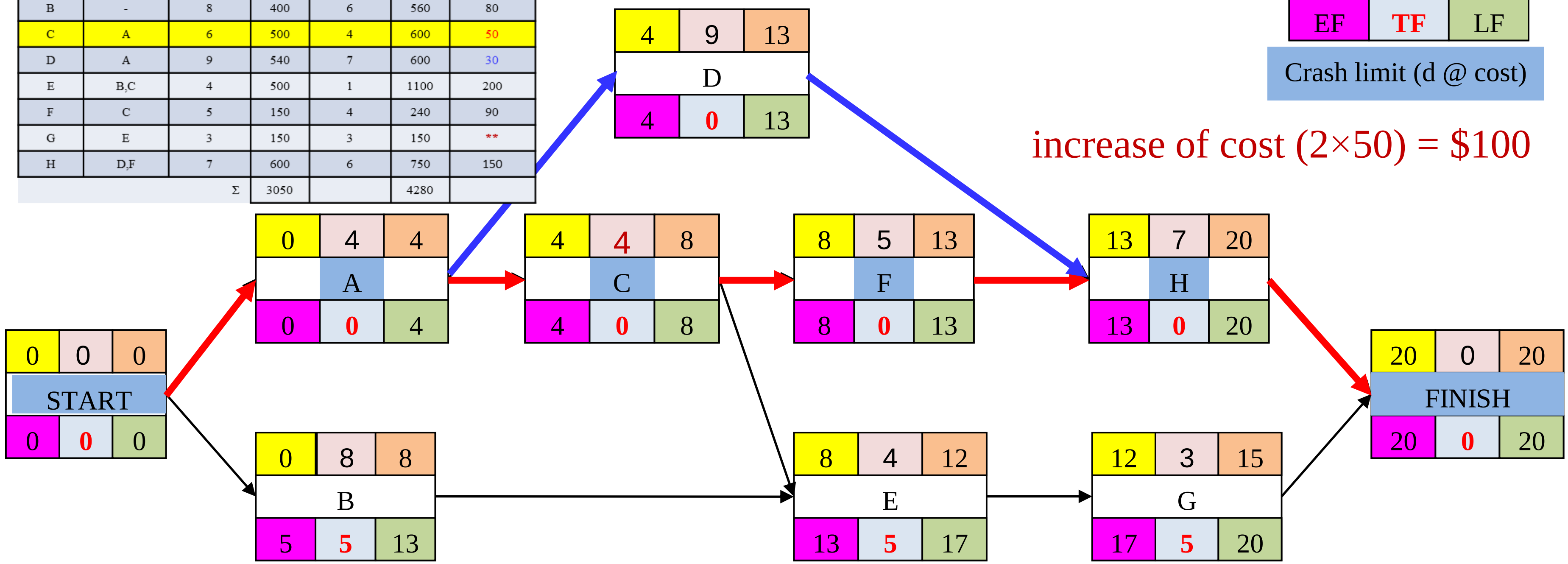
2- Among the critical activities the lowest slope is for activity C, so it can be expedited on critical path by 2 periods

# Step 4(a): Reduce 2 periods of activity C Solution-(i)



| Activity | Precedence | Normal | | Crash | | Cost Slope, |
|---|---|---|---|---|---|---|
| | | Time, day | Cost, $ | Time, day | Cost, $ | $/day |
| A | - | 4 | 210 | 3 | 280 | 70 |
| B | - | 8 | 400 | 6 | 560 | 80 |
| C | A | 6 | 500 | 4 | 600 | 50 |
| D | A | 9 | 540 | 7 | 600 | 30 |
| E | B,C | 4 | 500 | 1 | 1100 | 200 |
| F | C | 5 | 150 | 4 | 240 | 90 |
| G | E | 3 | 150 | 3 | 150 | ** |
| H | D,F | 7 | 600 | 6 | 750 | 150 |
| | Σ | | 3050 | | 4280 | |

increase of cost (2×50) = $100




| Activity | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| **Total float** | 0 | 5 | 0 | 0 | 5 | 0 | 5 | 0 |
| **Free float** | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 0 |

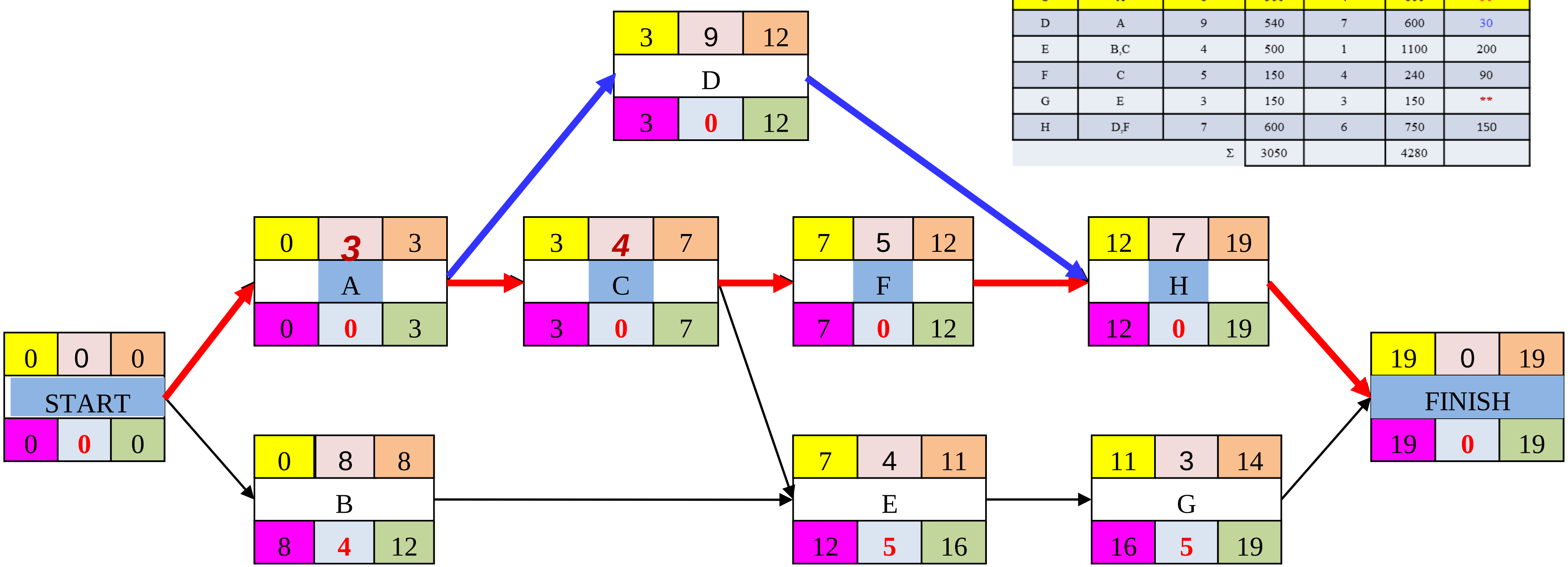
- Project completion time = 20 working days
- Critical Path: ***A, C, F, H.*** & ***A, D, H***

# Step 4(b): Reduce 1 period of activity A



Increase of cost = $70

| Activity | Precedence | Normal | | Crash | | Cost Slope, |
|---|---|---|---|---|---|---|
| | | Time, day | Cost, $ | Time, day | Cost, $ | $/day |
| A | - | 4 | 210 | 3 | 280 | 70 |
| B | - | 8 | 400 | 6 | 560 | 80 |
| C | A | 6 | 500 | 4 | 600 | 50 |
| D | A | 9 | 540 | 7 | 600 | 30 |
| E | B,C | 4 | 500 | 1 | 1100 | 200 |
| F | C | 5 | 150 | 4 | 240 | 90 |
| G | E | 3 | 150 | 3 | 150 | ** |
| H | D,F | 7 | 600 | 6 | 750 | 150 |
| | | Σ | 3050 | | 4280 | |




- Project completion time = 19 working days
- Critical Path: ***A, C, F, H.*** & ***A, D, H***

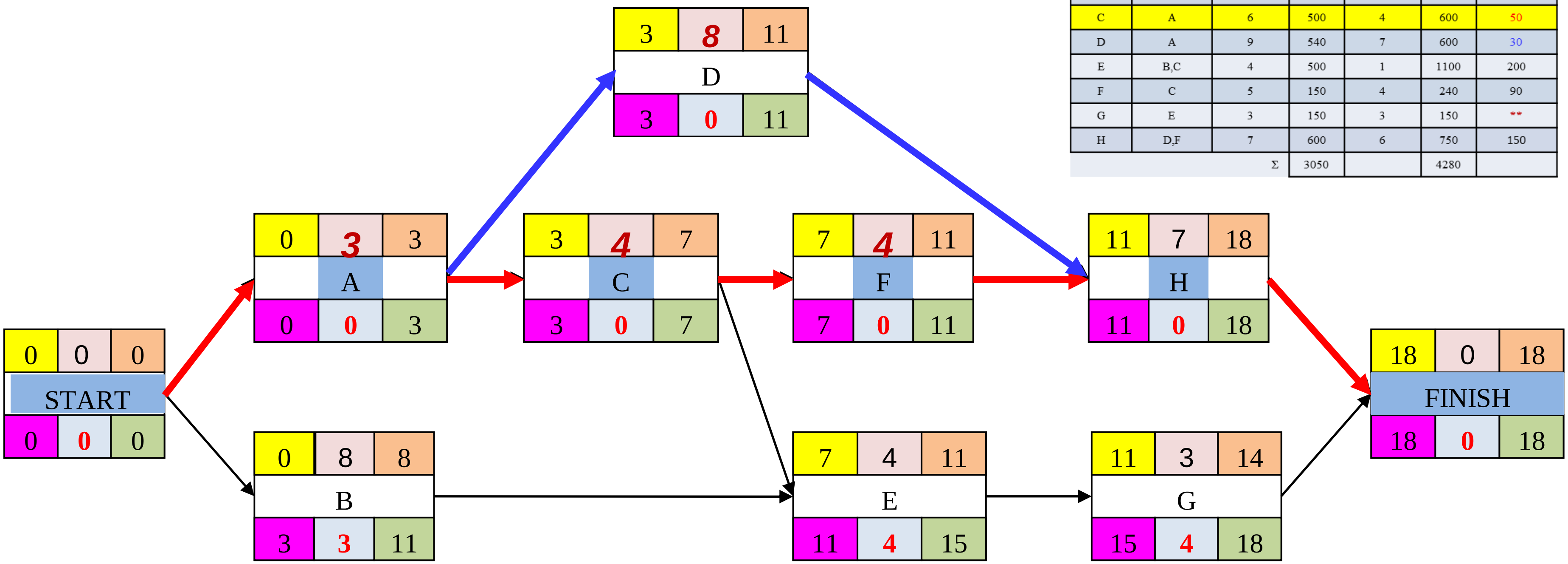
| Activity | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| Total float | 0 | 4 | 0 | 0 | 5 | 0 | 5 | 0 |
| Free float | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 0 |

# Step 4(c): Reduce 1 period of 2 activities (D,F)



**Increase of cost (30+90) = $120**

| Activity | Precedence | Normal | | Crash | | Cost Slope, |
|---|---|---|---|---|---|---|
| | | Time, day | Cost, $ | Time, day | Cost, $ | $/day |
| A | - | 4 | 210 | 3 | 280 | 70 |
| B | - | 8 | 400 | 6 | 560 | 80 |
| C | A | 6 | 500 | 4 | 600 | 50 |
| D | A | 9 | 540 | 7 | 600 | 30 |
| E | B,C | 4 | 500 | 1 | 1100 | 200 |
| F | C | 5 | 150 | 4 | 240 | 90 |
| G | E | 3 | 150 | 3 | 150 | ** |
| H | D,F | 7 | 600 | 6 | 750 | 150 |
| | Σ | | 3050 | | 4280 | |




- Project completion time = 18 working days
- Critical Path: ***A, C, F, H.*** & ***A, D, H***

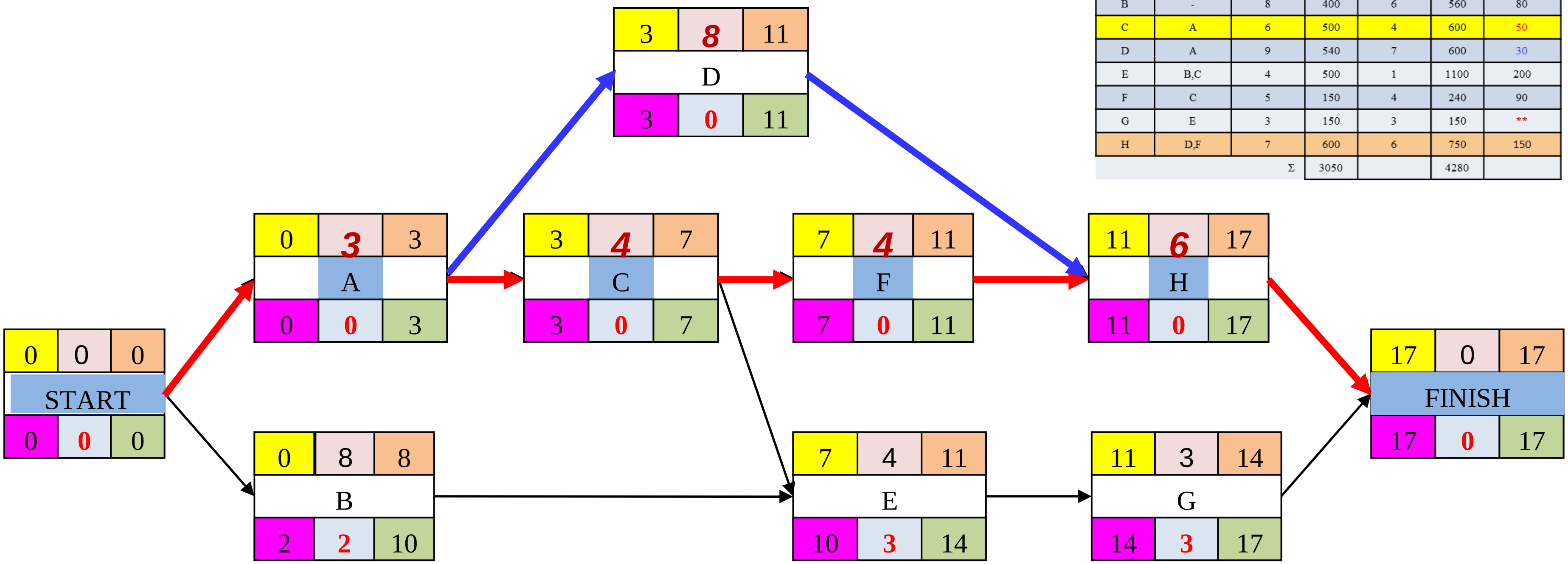
| Activity | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| **Total float** | 0 | 3 | 0 | 0 | 4 | 0 | 4 | 0 |
| **Free float** | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 |

# Step 4(d): Reduce 1 period of activity H



| Activity | Precedence | Normal | | Crash | | Cost Slope, |
|---|---|---|---|---|---|---|
| | | Time, day | Cost, $ | Time, day | Cost, $ | $/day |
| A | - | 4 | 210 | 3 | 280 | 70 |
| B | - | 8 | 400 | 6 | 560 | 80 |
| C | A | 6 | 500 | 4 | 600 | 50 |
| D | A | 9 | 540 | 7 | 600 | 30 |
| E | B,C | 4 | 500 | 1 | 1100 | 200 |
| F | C | 5 | 150 | 4 | 240 | 90 |
| G | E | 3 | 150 | 3 | 150 | ** |
| H | D,F | 7 | 600 | 6 | 750 | 150 |
| | | Σ | 3050 | | 4280 | |

**Increase of cost = $150**




- Project completion time = 17 working days
- Critical Path: ***A, C, F, H.*** & ***A, D, H***

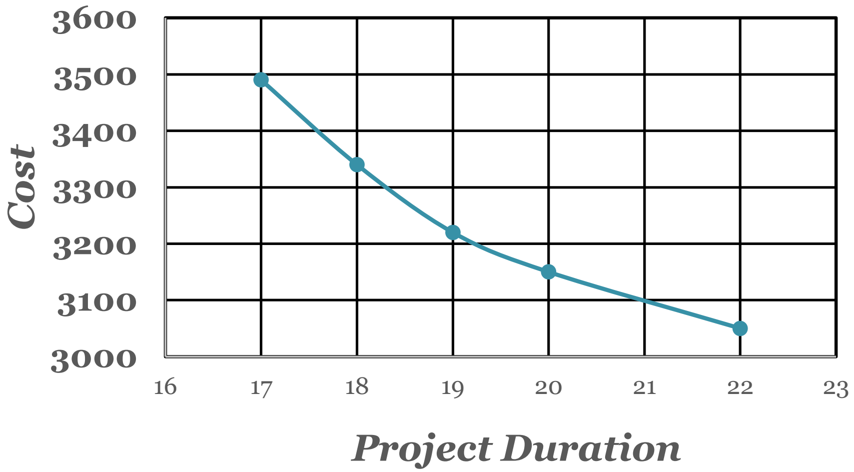
| Activity | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| Total float | 0 | 2 | 0 | 0 | 3 | 0 | 3 | 0 |
| Free float | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 |

# Step 5: Solution (ii)



| Cycle | Activity | Time | cost | Total cost | Duration |
|---|---|---|---|---|---|
| 0 | - | | | 3050 | 22 |
| 1 | C | 2 | 100 | 3150 | 20 |
| 2 | A | 1 | 70 | 3220 | 19 |
| 3 | D,F | 1 | 30+90 | 3340 | 18 |
| 4 | H | 1 | 150 | 3490 | 17 |

**Reduction of project duration from 22 to 17 days increases the cost to $3490.**




| Activity | Precedence | Normal | | Crash | | Cost Slope, |
|---|---|---|---|---|---|---|
| | | Time, day | Cost, $ | Time, day | Cost, $ | $/day |
| A | - | 4 | 210 | 3 | 280 | 70 |
| B | - | 8 | 400 | 6 | 560 | 80 |
| C | A | 6 | 500 | 4 | 600 | 50 |
| D | A | 9 | 540 | 7 | 600 | 30 |
| E | B,C | 4 | 500 | 1 | 1100 | 200 |
| F | C | 5 | 150 | 4 | 240 | 90 |
| G | E | 3 | 150 | 3 | 150 | ** |
| H | D,F | 7 | 600 | 6 | 750 | 150 |
| | Σ | | 3050 | | 4280 | |

# Network Interaction Limit (Nil)



$$\text{Nil} = \text{Min}\begin{cases} \text{Crash limit} \\ \text{Free Float of any of the non critical activities} \\ \text{in the parallel paths competing for critical path} \end{cases}$$

# Network Compression Algorithm



1. Determine normal project duration, cost and Critical Path
2. Compute the cost slope and shorten the Critical Activities beginning with the activity having the lowest cost-slope
3. Determine the compression limit (Nil), organize the data in the tabular form and update the project network
4. When a new Critical path is formed:
    1. Shorten the combination of activity which Falls on both Critical Paths, OR
    2. Shorten one activity from each of the critical paths. Use the combined cost of shortening both activities when determining if it is cost effective to shorten the project.
5. At each shortening cycle, compute the new project duration and project cost
6. Continue until no further shortening is possible
7. Tabulate and Plot the Indirect project Cost on the same time-cost graph; and add direct and indirect cost to find the project cost at each duration
8. Use the total project cost-time curve to find the optimum time

# Note



For large network, use criticality theorem to eliminate the noncritical paths that do not need to be crashed.

- Eliminate Activities with having TF > the required project reduction time.

# Problem-3



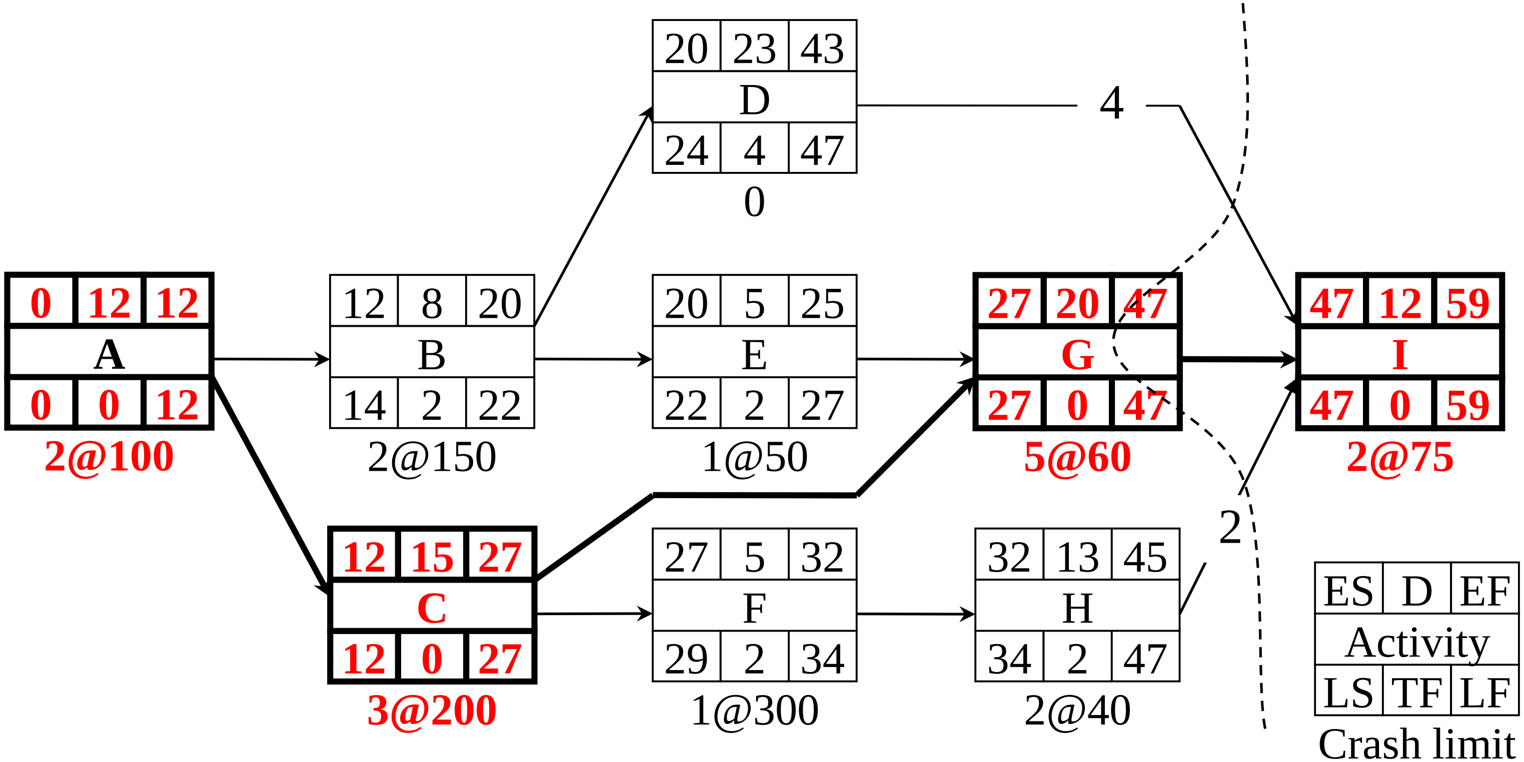
The durations and direct costs for each activity in the network of a small construction contract under both normal and crash conditions are given below. Establish the least cost for expediting the contract. Determine the optimum duration of the contract assuming the indirect cost amounts SR 125/week.

| Activity | Preceded by | Normal | | Crash | |
|---|---|---|---|---|---|
| | | Duration (weeks) | Cost (SR) | Duration (weeks) | Cost (SR) |
| A | — | 12 | 7000 | 10 | 7200 |
| B | A | 8 | 5000 | 6 | 5300 |
| C | A | 15 | 4000 | 12 | 4600 |
| D | B | 23 | 5000 | 23 | 5000 |
| E | B | 5 | 1000 | 4 | 1050 |
| F | C | 5 | 3000 | 4 | 3300 |
| G | E,C | 20 | 6000 | 15 | 6300 |
| H | F | 13 | 2500 | 11 | 2580 |
| I | D, G, H | 12 | 3000 | 10 | 3150 |
| | | | $\sum$36,500 | | 38470 |

# Solution

## Cycle #1

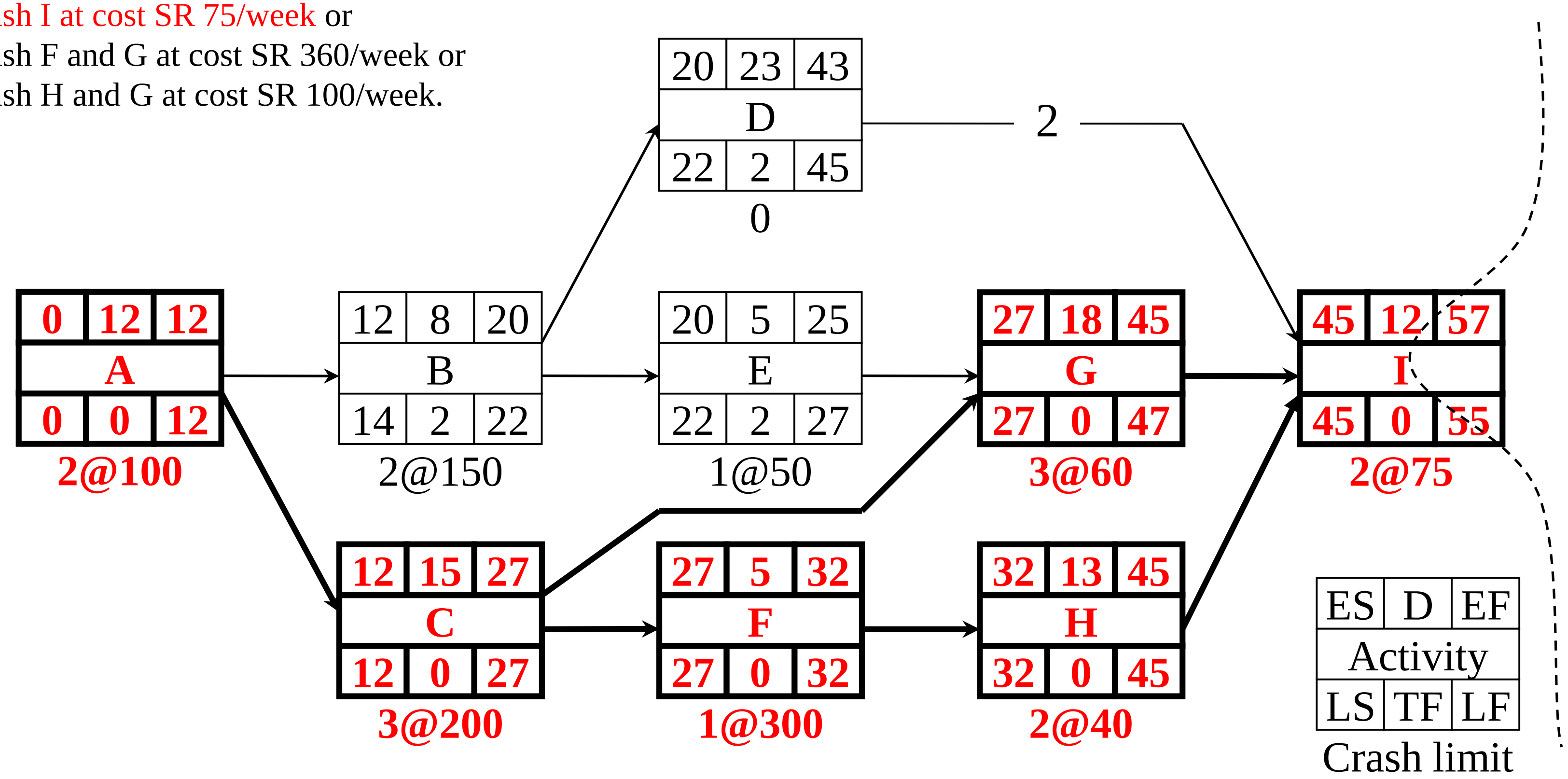
We have one critical path, A-C-G- I.
Either crash A at cost SR 100/week or
crash C at cost SR 200/week or
crash G at cost SR 60/week or
crash I at cost SR 75/week.




| Cycle # | Activity to Shorten | Can Be Shortened | Nil | Weeks Shortened | Cost per Week | Cost for Cycle | Total Cost | Project Duration |
|---|---|---|---|---|---|---|---|---|
| 0 | – | – | – | – | – | – | 36,500 | 59 |
| 1 | G | 5 | 2 | 2 | 60 | 120 | 36,620 | 57 |

## Cycle #2

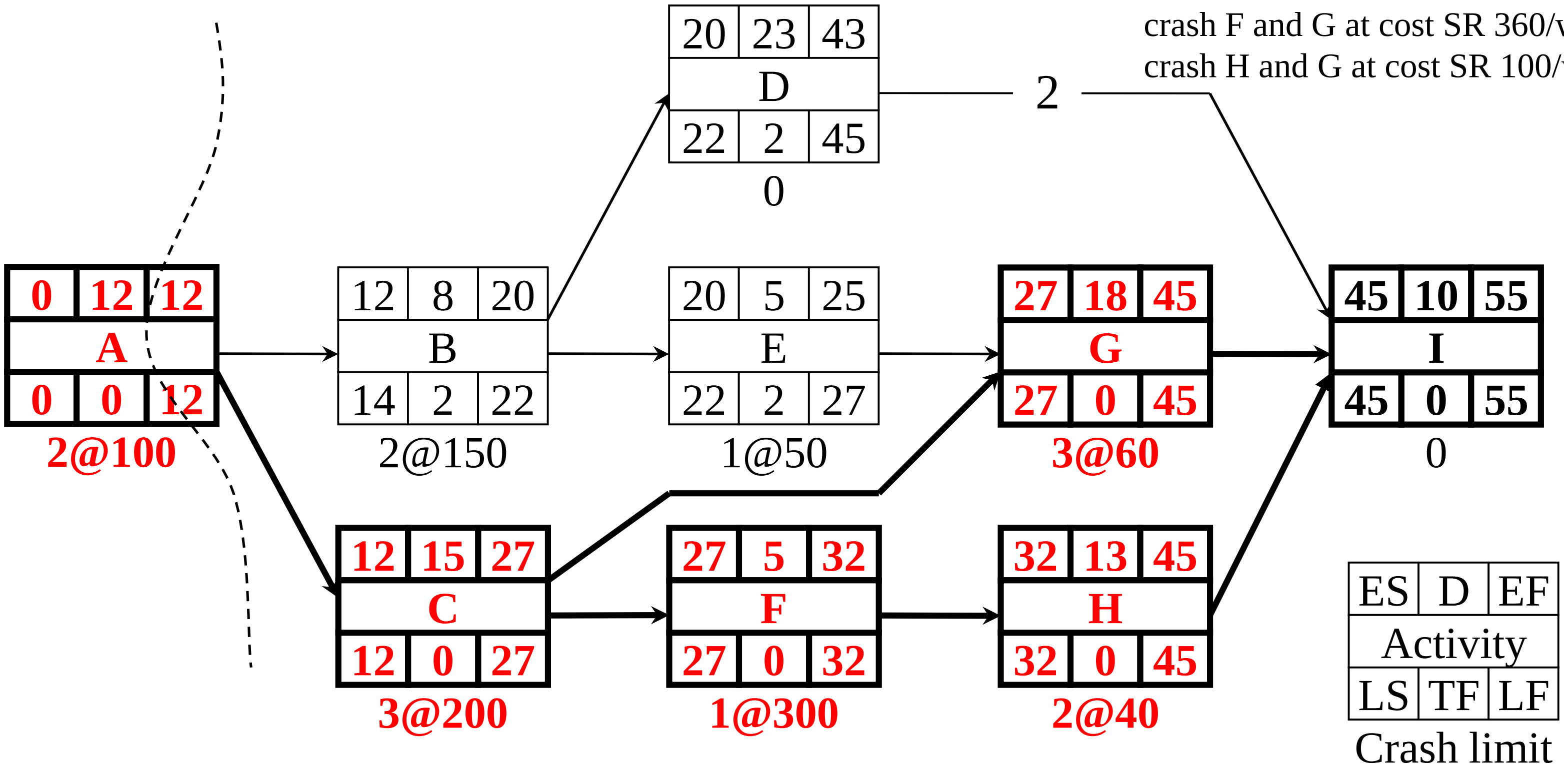
Now we have two critical paths:
A-C-F-H-I and A-C-G- I.
Either crash A at cost SR 100/week or
crash C at cost SR 200/week or
crash I at cost SR 75/week or
crash F and G at cost SR 360/week or
crash H and G at cost SR 100/week.




| Cycle # | Activity to Shorten | Can Be Shortened | Nil | Weeks Shortened | Cost per Week | Cost for Cycle | Total Cost | Project Duration |
|---|---|---|---|---|---|---|---|---|
| 0 | – | – | – | – | – | – | 36,500 | 59 |
| 1 | G | 5 | 2 | 2 | 60 | 120 | 36,620 | 57 |
| 2 | I | 2 | – | 2 | 75 | 150 | 36,770 | 55 |

# Cycle #3

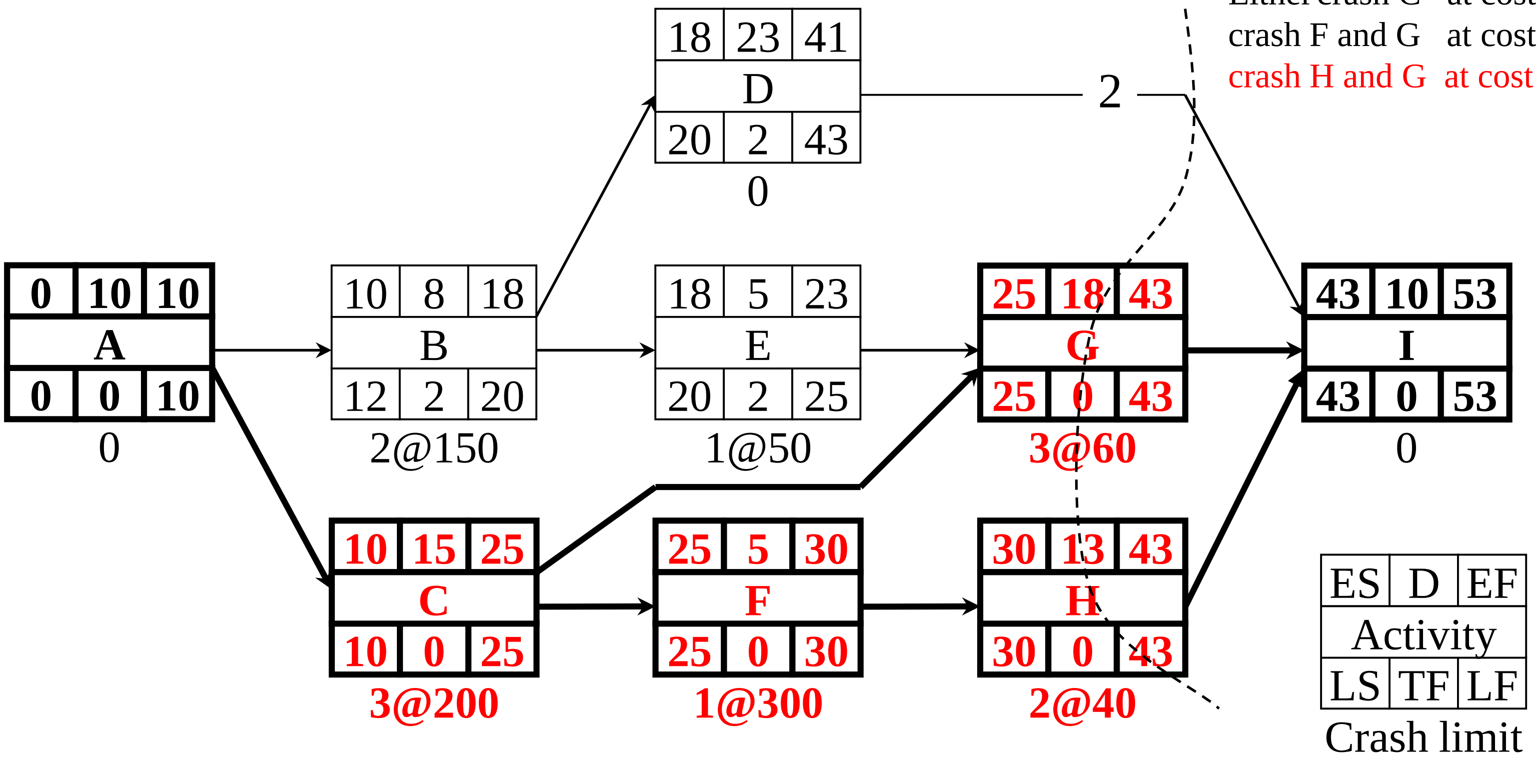
Now we have two critical paths:
A-C-F-H-I and A-C-G- I.
Either crash A at cost SR 100/week or
crash C at cost SR 200/week or
crash F and G at cost SR 360/week or
crash H and G at cost SR 100/week.




| Cycle # | Activity to Shorten | Can Be Shortened | Nil | Weeks Shortened | Cost per Week | Cost for Cycle | Total Cost | Project Duration |
|---|---|---|---|---|---|---|---|---|
| 0 | – | – | – | – | – | – | 36,500 | 59 |
| 1 | G | 5 | 2 | 2 | 60 | 120 | 36,620 | 57 |
| 2 | I | 2 | – | 2 | 75 | 150 | 36,770 | 55 |
| 3 | A | 2 | – | 2 | 100 | 200 | 36,970 | 53 |

# Cycle #4

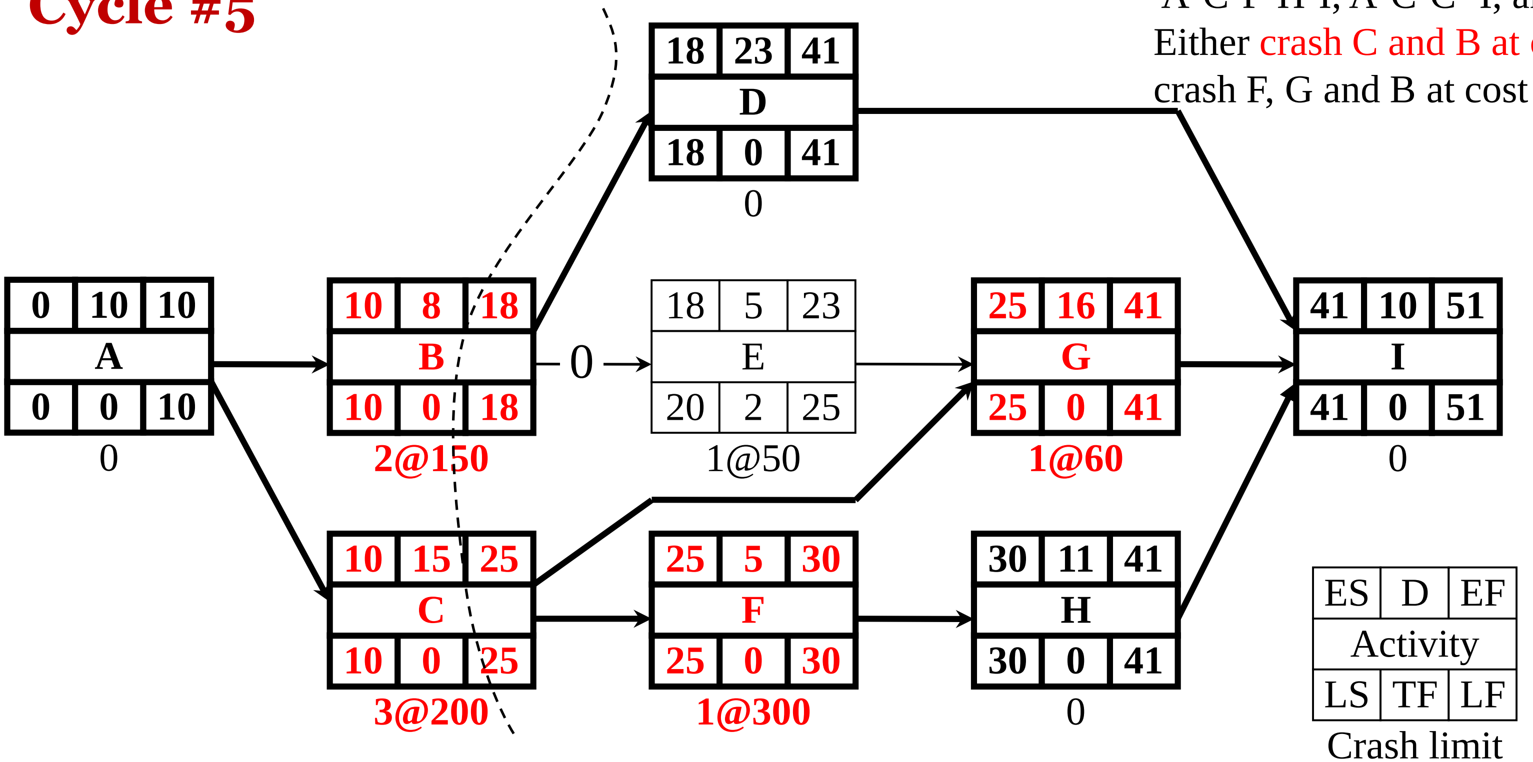
Now we have two critical paths:
A-C-F-H-I and A-C-G- I.
Either crash C at cost SR 200/week or
crash F and G at cost SR 360/week or
crash H and G at cost SR 100/week.




| Cycle # | Activity to Shorten | Can Be Shortened | Nil | Weeks Shortened | Cost per Week | Cost for Cycle | Total Cost | Project Duration |
|---|---|---|---|---|---|---|---|---|
| 0 | – | – | – | – | – | – | 36,500 | 59 |
| 1 | G | 5 | 2 | 2 | 60 | 120 | 36,620 | 57 |
| 2 | I | 2 | – | 2 | 75 | 150 | 36,770 | 55 |
| 3 | A | 2 | – | 2 | 100 | 200 | 36,970 | 53 |
| 4 | H, G | 2 | 2 | 2 | 60+40 | 200 | 37,170 | 51 |

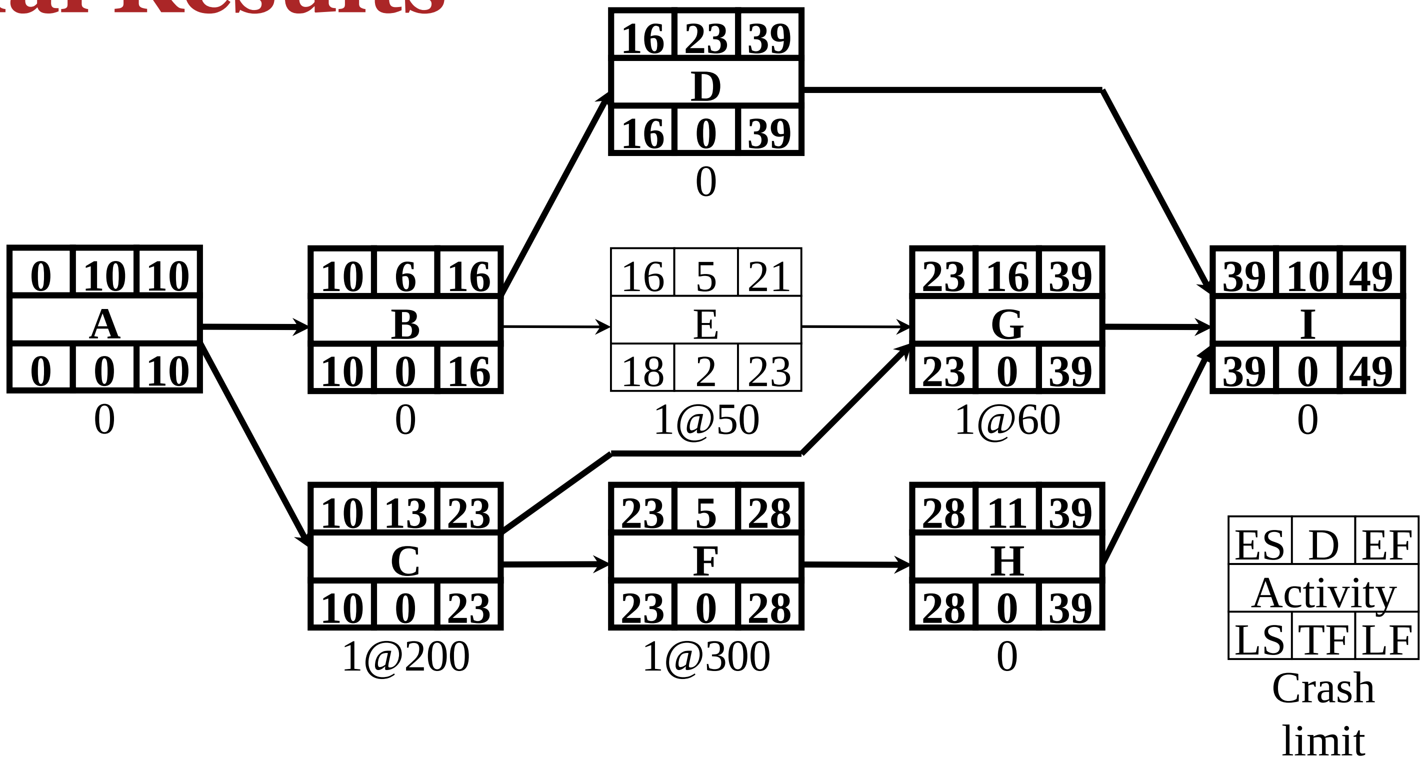
# Cycle #5

Now we have three critical paths;
A-C-F-H-I, A-C-C- I, and A-B-D- I.
Either crash C and B at cost SR 350/wk or crash F, G and B at cost SR 510/wk.




| Cycle # | Activity to Shorten | Can Be Shortened | Nil | Weeks Shortened | Cost per Week | Cost for Cycle | Total Cost | Project Duration |
|---|---|---|---|---|---|---|---|---|
| 0 | – | – | – | – | – | – | 36,500 | 59 |
| 1 | G | 5 | 2 | 2 | 60 | 120 | 36,620 | 57 |
| 2 | I | 2 | – | 2 | 75 | 150 | 36,770 | 55 |
| 3 | A | 2 | – | 2 | 100 | 200 | 36,970 | 53 |
| 4 | G, H | 2 | 2 | 2 | 60+40 | 200 | 37,170 | 51 |
| 5 | B, C | 2 | – | 2 | 150+200 | 700 | 37,870 | 49 |

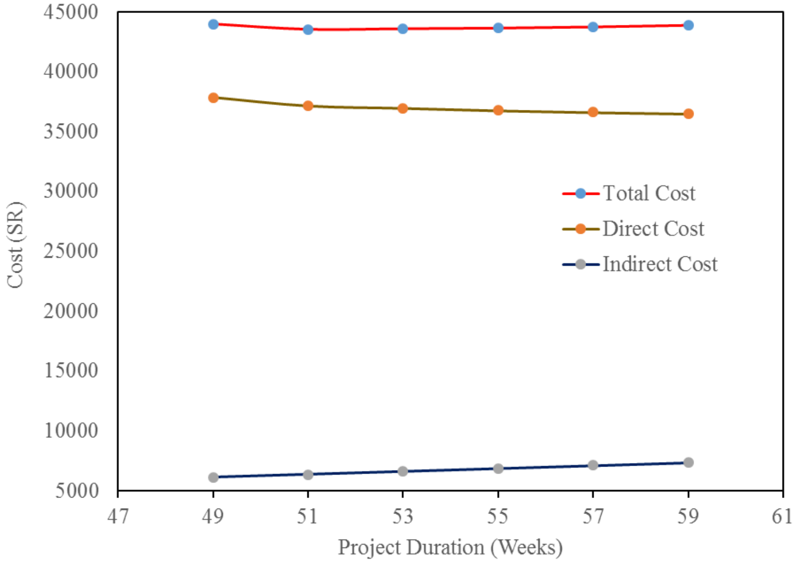
# Final Results

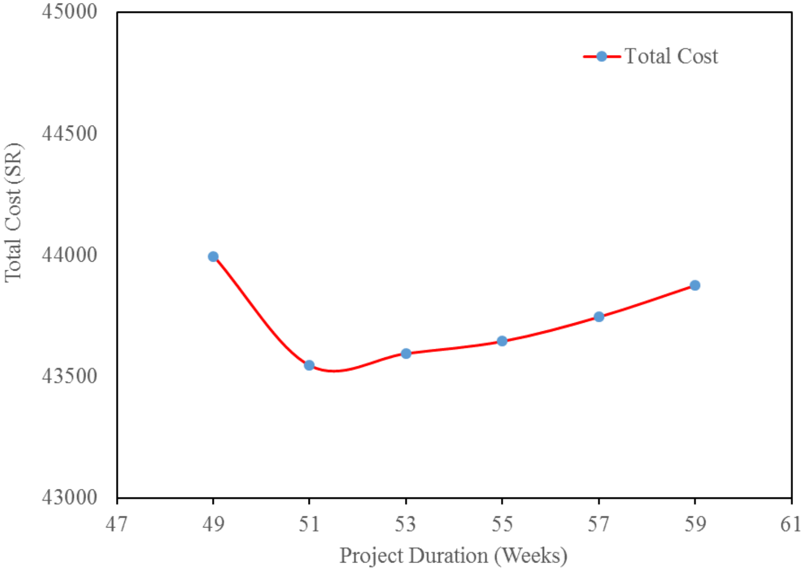



| Cycle # | Project Duration | Direct Cost | Indirect Cost | Total Cost |
|---|---|---|---|---|
| 0 | 59 | 36500 | 7375 | 43875 |
| 1 | 57 | 36620 | 7125 | 43745 |
| 2 | 55 | 36770 | 6875 | 43645 |
| 3 | 53 | 36970 | 6625 | 43595 |
| 4 | 51 | 37170 | 6375 | 43545 |
| 5 | 49 | 37870 | 6125 | 43995 |

# Project Optimal Duration





# Home Work



Data on small maintenance project is given as below: On completion, the project will give a return of SR110/day. Using time-cost trade-off method, how much would you like to compress the project for maximizing the return (ignore the Indirect cost effect)? Show all calculations.

| Activity | Depends on | Normal | | Crash | |
|---|---|---|---|---|---|
| | | Time | Cost | Time | Cost |
| A | – | 6 days | SR700 | 4 days | SR800 |
| B | – | 4 days | 400 | 4 days | 400 |
| C | – | 5 days | 650 | 4 days | 700 |
| D | A | 8 days | 625 | 5 days | 700 |
| E | B | 10 days | 200 | 7 days | 350 |
| F | B | 7 days | 500 | 5 days | 700 |
| G | C | 3 days | 600 | 3 days | 600 |
| H | D, E | 6 days | 300 | 5 days | 400 |
| I | F, G | 7 days | 350 | 4 days | 425 |

*Hint: If the project is compressed by n days, the return amount will be n*110 and total cost after compressing by n days will be obtained by the routine procedure. That n has to be searched which makes the difference of the above two costs maximum.*

# Further Reading



Read more about the Time-Cost Trade-offs from:

Jimmie W. Hinze. "Construction Planning and Management," Fourth Edition, 2012, Pearson.

# Thank You



## Questions Please