Department of Mathematics

M-106

First Mid Term Exam
Second Semester(1428/29)

Max. Marks: 20   **Time: 90 minutes**

Name:…………………………Number:…………..

Marks: **Multiple Choice(1-to-10)…………[   ]**

Question (11)…………[   ]

Question (12)…………[   ]

Question (13)…………[   ]

Question (14)………...[   ]

Total………………………….[   ]

Multiple Choices

Q.No.	1	2	3	4	5	6	7	8	9	10
{a,b,c,d}	a	b	b	a	a	a	b	b	b	b

Q.No:1 The sum $\sum_{k=1}^{10} k(k-2)^2$ is equal to:

    (a)   1705   (b)   1155 (c)   1255   (d) None of these.

Q.No:2 If $f(x) = 4x^3$, then the number z that satisfies the conclusion of the Mean Values theorem on [0,1] is

    (a)   1   (b) $\left(\dfrac{1}{4}\right)^{\frac{1}{3}}$ (c) $-\left(\dfrac{1}{4}\right)^{\frac{1}{3}}$ (d) None of these.

Q.No:3 The average value of the function $f(x) = 2x - x^2$ on the interval [0,1] is:

    (a) $\dfrac{1}{3}$ (b) $\dfrac{2}{3}$ (c)   1   (d) None of these.

Q.No:4 If $f(x) = \int_{x}^{x^2} Sin(t^2)dt$ then $f'(x)$ is equal to:

    (a) $2xSin(x^4) - Sin(x^2)$,   (b) $2xSin(x^4) + Sin(x^2)$,

                (c) $2xCos(x^4) - Cos(x^2)$,   (d) None of these.

Q .No:5 The value of the integral $\int_{1}^{2} \dfrac{e^{\frac{3}{x}}}{x^2} dx$ is equal to :

    (a) $e^{\frac{3}{2}} + e^3$, (b) $-\dfrac{1}{3}(e^{\frac{3}{2}} - e^3)$, (c) $\dfrac{1}{3}(e^3 - e^{\frac{3}{2}})$ (d) None of these.

Q.No:6 If $y = (x+1)^{Cos(x)}$, then y' at $x = 0$ is equal to

    (a)   1,   (b) -1,   (c) 2,   (d) None of these.

Q.No:7 The value of the integral $\int_{0}^{1} \dfrac{3x^2 + 2x}{x^3 + x^2 + 2} dx$ is equal to

    (a) $\ln(4)$,   (b) $\ln(2)$,   (c) $2\ln(4)$,   (d) None of these.

Q.No:8 If $\log_4(x^2) = 1$, then x is equal to:

    (a) 1 or -1,   (b) 2 or -2,   (c) 1 , 2,   (d) None of these.

Q.No:9 The value of the integral $\int \dfrac{Cosx}{1 + Sin^2x} dx$ is equal to

    (a) $\dfrac{1}{2}\ln(1 + Sin^2x)$, (b) $\tan^{-1}(Sin(x)) + c$, (c) $\tanh^{-1}(Sin(x)) + c$ (d) None of these.

Q.No:10 The value of the integral $\int \dfrac{Cosx}{1 - Sin^2x} dx$ is equal to

    (a) $-\dfrac{1}{2}\ln(1 - Sin^2x)$, (b) $\tanh^{-1}(Sin(x)) + c$, (c) $\tan^{-1}(Sin(x)) + c$ (d) None of these.

Question No: 11   Use the Simpson's rule to approximate the integral

$$\int_{-\pi/3}^{\pi/3} \tan(x)dx \;\; \text{with} \;\; n = 4.$$

Solution: $\Delta x = \dfrac{\pi/3 - (-\pi/3)}{4} = \dfrac{\pi}{6}.$ $x_0 = -\dfrac{\pi}{3}, x_1 = -\dfrac{\pi}{6}, x_2 = 0, x_3 = \dfrac{\pi}{6}, x_4 = \dfrac{\pi}{3}$

By Simpson's rule we have

$$\int_{-\pi/3}^{\pi/3} \tan(x)dx \approx \frac{\pi/3 - (-\pi/3)}{3(4)} \left\{ \tan(-\frac{\pi}{3}) + 4\tan(-\frac{\pi}{6}) + 2\tan(0) + 4\tan(\frac{\pi}{6}) + \tan(\frac{\pi}{6}) \right\}$$

$$= \frac{2\pi/3}{12} \left\{ -1.7 - 4(.57) + 2(0) + 4(.57) + (1.7) \right\} = 0.$$

Question No: 12   Evaluate the integral $\displaystyle\int_0^4 \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^3} dx$

Solution: Put $u = \sqrt{x} + 1$, $du = \dfrac{1}{2\sqrt{x}}dx$, $2du = \dfrac{1}{\sqrt{x}}dx$.

$$\int_0^4 \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^3} dx = \int_1^3 \frac{1}{u^3}(2du) = 2\left[-\frac{1}{2u^2}\right]_1^3 = 2\left[-\frac{1}{18}+\frac{1}{2}\right] = \frac{8}{9}$$

Question No: 13   Find y' if $y^2 + \ln\left(\dfrac{x}{y}\right) - 4x = -3$.

Solution: $y^2 + \ln(x) - \ln(y) - 4x = -3$.
            Now on differentiating with respect to x we get:

$$2yy' + \frac{1}{x} - \frac{1}{y}y' - 4 = 0 \;.\; \left(2y - \frac{1}{y}\right)y' = 4 - \frac{1}{x}.$$

$$y' = \frac{\left(4 - \frac{1}{x}\right)}{\left(2y - \frac{1}{y}\right)}.$$

Question No: 14   Evaluate the integral $\int \dfrac{1}{x \ \sqrt{x^8 - 16}} dx$.

Solution: $\int \dfrac{1}{x \ \sqrt{x^8 - 16}} dx = \int \dfrac{1}{x \ \sqrt{(x^4)^2 - (4)^2}} dx$

Now put $u = x^4 \ \Rightarrow \ du = 4x^3 dx , \ \dfrac{1}{4} du = x^3 dx$

$\int \dfrac{1}{x \ \sqrt{(x^4)^2 - (4)^2}} dx = \int \dfrac{x^3}{x^4 \ \sqrt{(x^4)^2 - (4)^2}} dx$ (multiplied and divide by x^3)

$\qquad = \int \dfrac{1}{u\sqrt{(u)^2 - (4)^2}} \dfrac{1}{4} du \ = \ \dfrac{1}{4} \int \dfrac{1}{u\sqrt{(u)^2 - (4)^2}} du$

$\qquad = \dfrac{1}{4} \left[\dfrac{1}{4} Sec^{-1} \left(\dfrac{u}{4} \right) \right] + c \ = \ \dfrac{1}{16} Sec^{-1} (x^4) + c$