

106Midterm1 solutions(Sem1-37/38)

Question1 a) $F'(x) = \cosh x \cdot \int_0^{2x} (1+t^2)^5 dt + 2\sinh x(1+4x^2)^5$ (1)

$$F'(0) = 0 \quad (1)$$

b) $\int_0^{\pi/4} \frac{(1+\tan x)^6}{\cos^2 x} dx = \int_0^{\pi/4} (1+\tan x)^6 \sec^2 x dx$ (1)

$$= \int_1^2 u^6 du \quad u = 1 + \tan x, du = \sec^2 x dx \quad (1)$$
$$= [u^7/7]_1^2 = \frac{1}{7}(2^7 - 1) \quad (1)$$

c) $\int_{-1}^2 \sqrt{x+2} dx = \frac{2}{3} [(x+2)^{3/2}]_{-1}^2 = \frac{14}{3}$ (1)

$$\frac{14}{9} = \sqrt{c+2} \quad (1) \quad \text{so } c = \frac{34}{81} \quad (1)$$

Question2 a) $x_0 = 0, x_1 = \frac{1}{4}, x_2 = \frac{1}{2}, x_3 = \frac{3}{4}, x_4 = 1$ (1)

$$T_4 = \frac{1}{8} \left(1 + 2\sqrt{\frac{17}{16}} + 2\sqrt{\frac{5}{4}} + 2\frac{5}{4} + \sqrt{2} \right) \quad (1)$$

$$T_4 \approx \frac{1}{8}(1 + 2.061553 + 2.236068 + 2.5 + 1.414213) \approx 1.151479$$

(1)

b) $f'(x) = \frac{(\cosh^{-1}(2x))'}{\ln 10 \cdot \cosh^{-1}(2x)} = \frac{2}{\ln 10 \cdot \cosh^{-1}(2x) \cdot \sqrt{4x^2 - 1}}$ (1) + (1)

Question3 a) $y = \frac{x^x \cdot \sqrt[3]{1+4x}}{\sin^{-1} x}$

$$\ln y = x \ln x + \frac{1}{3} \ln(1+4x) - \ln(\sin^{-1}(x)) \quad (1)$$

$$\frac{y'}{y} = \ln x + 1 + \frac{4}{3(1+4x)} - \frac{1}{\sin^{-1}(x) \cdot \sqrt{1-x^2}} \quad (1,5)$$

$$y' = \left(\ln x + 1 + \frac{4}{3(1+4x)} - \frac{1}{\sin^{-1}(x) \cdot \sqrt{1-x^2}} \right) y \quad (0,5)$$

$$\text{b) } \int \frac{2x \ln(1+x^2) dx}{1+x^2} = \int u du \quad u = \ln(1+x^2) \quad du = \frac{2x dx}{1+x^2} \quad (2)$$

$$= \frac{u^2}{2} + C = \frac{1}{2} (\ln(1+x^2))^2 + C \quad (1)$$

Question3

$$\begin{aligned} \text{a) } \int \frac{3^x dx}{2+3^{2x}} &= \frac{1}{\ln 3} \int \frac{du}{2+u^2} \quad u = 3^x, \quad du = \ln 3 \cdot 3^x dx \quad (2) \\ &= \frac{1}{\sqrt{2} \ln 3} \tan^{-1} \left(\frac{u}{\sqrt{2}} \right) + C = \frac{1}{\sqrt{2} \ln 3} \tan^{-1} \left(\frac{3^x}{\sqrt{2}} \right) + C \\ &\quad (1) \end{aligned}$$

$$\begin{aligned} \text{b) } \int \frac{dx}{\sqrt{9-e^{6x}}} &= \frac{1}{3} \int \frac{du}{u\sqrt{3^2-u^2}} \quad u = e^{3x} \quad dx = \frac{du}{3u} \quad (2) \\ &= -\frac{1}{9} \operatorname{sech}^{-1} \left(\frac{e^{3x}}{3} \right) + C \quad (1) \end{aligned}$$