KING SAUD UNIVERSITY
College of Science
Department of Mathematics

First Semester (1433/1434)

First Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 25 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a, b, c, d\}$	c	b	a	a	b	b	a	c	a	a

Q. No: 1 $\frac{d}{dx}\int_{-x}^{1} f(t)\, dt$ equals to:

(a) $f(x)$ (b) $-f(x)$ (c) $f(-x)$ (d) $-f(-x)$

Q. No: 2 The value of the integral $\int_{1}^{4} (2 + 3\sqrt{x})\, dx$ is equal to:

(a) 19 (b) 20 (c) 21 (d) 22

Q. No: 3 By using the integral $\int_{0}^{3} (4 - \frac{x^2}{4}) dx = \frac{39}{4}$. Then the number z that satisfies the conclusion of the Mean value Theorem for this integral will be:

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{3}{2}$ (d) $\sqrt{\frac{3}{2}}$

Q. No: 4 The sum $\sum_{k=1}^{n} \left(3k^2 - \frac{1}{2}\right)$ is equal to:

(a) $\frac{1}{2}n^2(2n+3)$ (b) $\frac{1}{2}n^2(2n+2)$

(c) $\frac{1}{2}n^2(2n+1)$ (d) $\frac{1}{3}n^2(2n+1)$

Q. No: 5 Find x such that $x = \log(1) - \log(10) + 1$

(a) $\simeq -1.3026$ (b) 0 (c) 1 (d) 2

Q. No: 6 The derivative of the function $f(x) = \tanh^{-1}(x^2 - 1)$ is equal to:

(a) $\frac{x}{x^2 - x^4}$ (b) $\frac{2x}{2x^2 - x^4}$ (c) $\frac{2x}{x^2 - 2x^4}$ (d) $\frac{2x}{x^2 - 4x^4}$

Q. No: 7 Find $\int \sec(x)\left[\sec(x) + \tan(x)\right] dx$:

(a) $\tan(x) + \sec(x) + c$ (b) $\tan(x) + \csc(x) + c$

(c) $\cot(x) + \csc(x) + c\ + c$ (d) $\cot(x) + \sec(x) + c$

Q. No: 8 Assume that $\dfrac{dy(x)}{dx} = \sin(x+2)$, then:

$(a)\ y(x) = \dfrac{1}{2}\cos(x+2) + c \quad (b)\ y(x) = -\dfrac{1}{2}\cos(x+2) + c$

$(c)\ \ y(x) = -\cos(x+2) + c \quad (d)\ y(x) = \cos(x+2) + c$

Q. No: 9 The integral $\displaystyle\int \frac{1}{\sqrt{1-e^{2x}}}dx$ is equal to:

$(a)\ -\operatorname{sech}^{-1}(e^x) + c \quad (b)\ \operatorname{sech}^{-1}(e^x) + c$

$(c)\ \ -\cosh^{-1}(e^x) + c \quad (d)\ \ \cosh^{-1}(e^x) + c$

Q. No: 10 The integral $\displaystyle\int x2^{x^2}dx$ is equal to:

$(a)\ \dfrac{1}{2}\dfrac{2^{x^2}}{\ln 2} + c \qquad (b)\ \dfrac{2^{x^2}}{\ln 2} + c \qquad (c)\ \dfrac{1}{2}\dfrac{2^{x^2}}{x\ln 2} + c \qquad (d)\ \dfrac{2^{x^2}}{4} + c$

Full Questions

Question No: 11 Approximate the integral $\int_1^2 \frac{1}{\sqrt{3+x^2}}dx$ using the **Trapezoidal rule** for a regular partition with $n = 4$. [**3**]

Answer: We have $[a,b] = [1,2]$, $n = 4$ and $f(x) = \frac{1}{\sqrt{3+x^2}}$. Then

$$\Delta x = \frac{b-a}{4} = \frac{2-1}{4} = 0.25, \ [0.5]$$

and also we can get

$$x_0 = 1, \ x_1 = 1.25, \ x_2 = 1.5, \ x_3 = 1.75, \quad x_4 = 2, \ [0.5]$$

Then

$$\begin{aligned}\int_1^2 \frac{1}{\sqrt{3+x^2}}dx &\simeq \frac{2-1}{2(4)}[f(1) + 2f(1.25) + 2f(1.5) + 2f(1.75) + f(2)], \ [1] \\ &\simeq \frac{0.5 + 2(0.468165) + 2(0.436436) + 2(0.406138) + 0.377964}{8}, \quad [0.5] \\ &\simeq \frac{3.49944}{8} \simeq 0.43743. \qquad [0.5]\end{aligned}$$

Question No: 12 If $y(x) = (x^4 + x^2 + 1)^{\ln(x^2+1)}$, then find $y'(x)$. [**4**]

Answer: We have

$$\ln(y(x)) = \ln(x^2+1)\ln\left(x^4+x^2+1\right), \quad [1]$$

Then

$$\begin{aligned}\frac{y'(x)}{y(x)} &= \ln\left(x^4+x^2+1\right)\frac{2x}{x^2+1} + \frac{4x^3+2x}{x^4+x^2+1}\ln(x^2+1), \ [2] \\ y'(x) &= y(x)\left[\ln\left(x^4+x^2+1\right)\frac{2x}{x^2+1} + \frac{4x^3+2x}{x^4+x^2+1}\ln(x^2+1)\right], \ [0.5] \\ &= \left(x^4+x^2+1\right)^{\ln(x^2+1)}\left[\ln\left(x^4+x^2+1\right)\frac{2x}{x^2+1} + \frac{4x^3+2x}{x^4+x^2+1}\ln(x^2+1)\right], \ [0.5]\end{aligned}$$

Question No: 13 Evaluate the integral $\int \frac{1}{x\sqrt{x^8-25}}dx$. [**4**]

Answer: If we denote by:

$$u = x^4, \quad [1]$$

then

$$du = 4x^3 dx \text{ then } dx = \frac{du}{4x^3}, \ [0.5]$$

$$\begin{aligned} \int \frac{1}{x\sqrt{x^8-25}}dx &= \int \frac{1}{4u\sqrt{u^2-25}}du = \frac{1}{4}\int \frac{1}{u\sqrt{u^2-(5)^2}}du, \ [0.5] \\ &= \frac{1}{20}\sec^{-1}(\frac{u}{5}) + c = \frac{1}{20}\sec^{-1}(\frac{x^4}{5}) + c. \ [2] \end{aligned}$$

Question No: 14 If $\int_1^x f(t)dt = x\ln(x)$, calculate $f\left(e^2\right)$?. [**4**]

Answer:

$$\begin{aligned} \frac{d}{dx}\int_1^x f(t)dt &= \frac{d}{dx}(x\ln(x)), \ [1] \\ f(x) &= \ln(x) + 1, \ [2] \end{aligned}$$

then

$$f\left(e^2\right) = \ln(e^2) + 1 = 2\ln(e) + 1 = 2(1) + 1 = 3. \ [1]$$