King Saud University
Department of Mathematics

Final Exam

1^{st} **semester 1437 H**

Course Title: Math 244 (Linear Algebra)

Date: Tuesday 5 January 2016; (8-11) am

(. . . . . .) Name . . . . ID . . . . Section . . . .

Lecturer . . . .

Question	Grade
Q1	
Q2	
Q3	
Q4	
Total	

Part I	(a)	(b)	(c)	(d)	(e)	(f)
Answer						

Question 1

I. Choose the correct answer (write it down on the table above):

(a) The dimension of $M_{2\times 2}(R)$, the vector space of all 2×2 matrices of real numbers, is

(i) 2
(ii) 4
(iii) 0
(iv) None.

(b) The reflection of $(2, -5, 3)$ about the $yz - plane$ is

(i) $(2, 5, 3)$
(ii) $(-2, -5, 3)$
(iii) $(-2, -5, -3)$
(iv) None.

(c) The nullity of $\begin{bmatrix} 1 & -3 & 2 & 5 \\ -2 & 6 & 0 & -3 \\ 4 & -12 & -4 & -1 \end{bmatrix}$ is

(i) 1
(ii) 3
(iii) 2
(iv) None

(d) $T_1 \circ T_2$ in R^3 where T_1 is the rotation of $90°$ about the $y - axis$ and T_2 is a reflection about the $xz - plane$ is

(i) $\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
(ii) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$
(iii) $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$
(iv) None.

(e) If $u = (1, 2, -3)$; $v = (3, -3, 5)$ and $w = (0, -1, -3)$ then $(u \cdot v) - w$ is

(i) 22

(ii) $(18, 17, 15)$

(iii) $(18, 19, 21)$

(iv) None

(f) Let $T_A : R^3 \to R^3$ be multiplication by $A = \begin{bmatrix} 4 & -1 & -2 \\ 5 & 1 & 2 \\ -3 & 6 & -4 \end{bmatrix}$ and let e_1, e_2 and e_3 be the standard basis vectors for R^3, then $T_A(e_1 + e_2 + e_3)$ is

(i) $\begin{bmatrix} 1 \\ 8 \\ -1 \end{bmatrix}$

(ii) $\begin{bmatrix} -1 \\ 8 \\ 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$

(iv) None

II. Determine wether the following is true or false.

(a) If $V = span\{v_1, v_2, \cdots, v_n\}$, then $\{v_1, v_2, \cdots, v_n\}$ is a basis for V. [ ]

(b) If $S = \{v_1, v_2, v_3\}$ is a linearly independent set in R^3 then it is a basis for R^3. [ ]

(c) If R is the reduced echelon form of a matrix A, then those column vectors of R that contain the leading $1s'$ form a basis for the column space of A. [ ]

(d) The system $A\mathbf{x} = \mathbf{b}$ is inconsistent if and only if $\mathbf{b}$ is not in the column space of A. [ ]

(e) If $A = [a_{ij}]_{n\times n}$ and $T_A : R^n \to R^n$ is the corresponding matrix operator then T_A is one-to-one if and only if the range of T_A is R^n. [ ]

(f) The eigenvalues of a matrix A are the same as the eigenvalues of the row echelon form of A. [ ]

(g) For every $A = [a_{ij}]_{n\times n}$, $A \cdot adj(A) = \big(\det(A)\big)I$. [ ]

(h) If A^2 is a symmetric matrix then A is a symmetric matrix. [ ]

(i) If the reduced row echelon form of an augmented matrix for a linear system has a row of zeros, then the system must have infinitely many solutions. [ ]

(j) $W = \{(x, y) \in R^2, x^2 = y^2\}$ is a subspace of R^2. [ ]

Question 2

(a) (i) If A is a nonsingular (invertible) matrix whose inverse is $\begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$, find A.

(ii) Show that $p(t) = -t^2 + t - 4$ belongs to the $span\{t^2 + 2t + 1,\ t^2,\ t - 1\}$.

(iii) Find the row and column rank of $A = \begin{bmatrix} 1 & 1 & -1 & 2 & 0 \\ 2 & -4 & 0 & 1 & 1 \\ 5 & -1 & -3 & 7 & 1 \\ 3 & -9 & 1 & 0 & 2 \end{bmatrix}$.

(b) (i) If $W = \{X \in M_{2\times 2}(R) : AX = XA\} \subseteq M_{2\times 2}(R)$, where $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$. Show that W is a subspace of $M_{2\times 2}(R)$.

(ii) (1.) Find a subset of the vectors that forms a basis for the space spanned by the vectors: $v_1 = (1, -2, 0, 3)$; $v_2 = (2, -4, 0, 6)$; $v_3 = (0, -1, 2, 3)$.

(2.) Express each vector that is not in the basis as a linear combination of the basis vectors.

Question 3

(a) Let

$$A = \begin{bmatrix} 7 & 0 & -3 \\ -9 & -2 & 3 \\ 18 & 0 & -8 \end{bmatrix}$$

(i) Compute the eigenvalues of A.

(ii) Find the bases for the eigenspaces of A.

(b) (i) Show that the matrix operator $T : R^3 \to R^3$ defined by the equations:

$$\begin{aligned} x_1 - 2x_2 + 2x_3 &= w_1 \\ 2x_1 + x_2 + x_3 &= w_2 \\ x_1 + x_2 &= w_3 \end{aligned}$$

is one-to-one.

(ii) Find the standard matrix for the inverse operator.

(iii) Find $T^{-1}(w_1, w_2, w_3)$.

Question 4 5 marks Bonus

1. (i) Show that the vectors $u = (1, -5, 4)$ and $v = (3, 3, 3)$ are orthogonal.

   (ii) Verify the theorem of Pythagoras $\|u + v\|^2 = \|u\|^2 + \|v\|^2$ for u and v.

2. Let $C = \begin{bmatrix} 4 & 3 \\ 0 & -2 \end{bmatrix}$. Find the eigenvalues of C^3.

3. Find the vector $v = (v_1, v_2, v_3)$ in R^3 whose coordinate vector relative to the basis

$$S = \{(3, 2, 1), (-2, 1, 0), (5, 0, 0)\}$$

   is $(v)_S = (-1, 3, 2)$.