King Saud University
Department of Mathematics

151
Final Exam, January 2015

NAME:

Group Number:

ID:

Question	Grade
I	
II	
III	
IV	
V	
VI	
Total	

Question	1	2	3	4	5	6	7	8	9	10	11
Answer											

I) Choose the correct answer (write it on the table above):

1) The proposition $(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$ is

(A) a tautology	(B) a contradiction	(C) None of the previous

2) The argument

p
$p \to q$
$\neg q \vee r$
$-----$
r
is

(A) valid	(B) invalid

3) The statement $\neg\exists x(\neg p(x) \wedge q(x))$ is logically equivalent to

(A) $\exists x(p(x) \vee \neg q(x))$	(B) $\forall x(p(x) \vee \neg q(x))$	(C) $\forall x(\neg p(x) \wedge q(x))$	(D) None of the previous

4) An equivalent expression for the statement $\exists x \in \mathbb{R}$ such that $x^2 = 2$ is

(A) The square of each number is 2	(B) If x is a real number, then $x^2 = 2$	(C) There is at least one real number whose square is 2	(D) None of the previous

5) In the congruence relation modulo 5 ($\equiv \mod 5$) on $\mathbb{Z}$

(A) $3 \in [2]$	(B) $7 \in [0]$	(C) $-9 \in [1]$	(D) None of the previous

6) If R is an equivalence relation and xRy, then

(A) $[x] \cap [y] = \emptyset$	(B) $[x] = [y]$	(C) $[x] \neq [y]$	(D) None of the previous

7) Which of the following relations is false?

(A) $\emptyset \subseteq \mathbb{Z}$	(B) $\mathbb{Q} \not\subseteq \mathbb{Z}$	(C) $\mathbb{Q} \subseteq \mathbb{Z}$	(D) None of the previous

8) The number of edges of the graph $K_{4,5}$ is

(A) 9	(B) 40	(C) 20	(D) None of the previous

9) A graph with 4 vertices, each of degree 2 has

(A) 6 edges	(B) 4 edges	(C) 8 edges	(D) None of the previous

10) The grapg C_3 is

(A) bipartite	(B) not connected	(C) not bipartite	(D) None of the previous

11) If $f(x, y, z) = xy + y + \overline{xz}$ is a Boolean function, then $f(0, 1, 0)$ equals

(A) 0	(B) 1	(C) None of the previous

II) A) Prove (by cases) that, for any integer n, the product $n(n+1)$ is even.

B) A sequence $(a_n)_{n \geq 1}$ is defined by $a_1 = 3$ and $a_n = 7a_{n-1}$ for $n \geq 2$. Prove that $a_n = 3 \cdot 7^{n-1}$, for all $n \geq 1$.

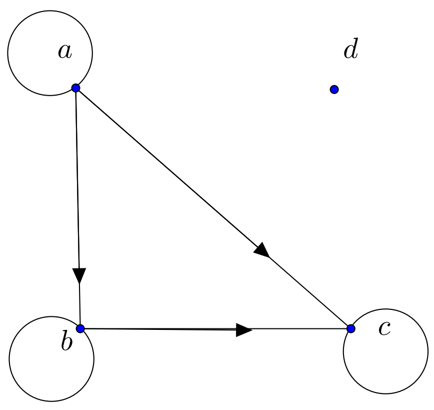
III) A) On $\mathbb{Z} \times \mathbb{Z}$, define the relation R through

$$(a, b)R(c, d) \Longleftrightarrow a \leq c \quad \text{and} \quad b \leq d.$$

i) Prove that R is a partial order relation;

ii) Is R a total order relation? Justify your answer.

B) A relation R on the set $\{a, b, c, d\}$ is represented by the diagraph below:



i) List the ordered pair in the relation R;

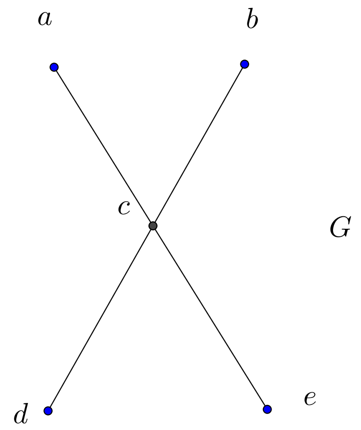
ii) Is the relation R reflexive? Justify your answer;

iii) Is the relation R transitive? Justify your answer;

iv) Is the relation R symmetric? Justify your answer;

v) Is the relation R antisymmetric? Justify your answer.

IV) A) Let G be the graph below:



i) Is the graph G connected? Justify your answer;

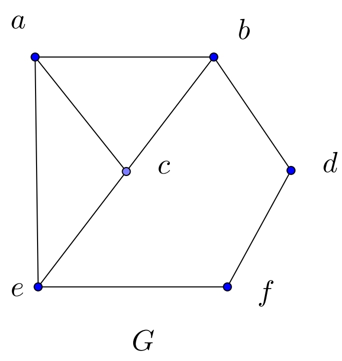
ii) Find $\deg(e)$;

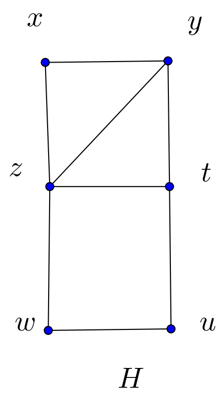
iii) Find a path from a to b. What is its length?

iv) Is the graph G a subgraph of the wheel W_4?

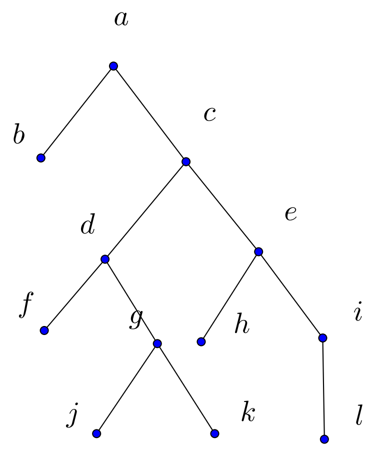
v) Is the graph G bipartite? Justify your answer.

B) Are the two graphs G and H, represented below, isomorphic? Justify.





V) Consider the tree below:



i) Which vertex is the root?

ii) List the internal vertices;

iii) List the leaves;

iv) What is the parent of e?

v) What are the siblings of c?

vi) Is this tree a binary tree? Justify your answer;

vii) Find the level of each vertex of the tree;

viii) What is the height of the tree? Justify your answer.

VI) A) Consider the Boolean function

$$F(x, y, z) = \overline{x} \cdot y + \overline{x} \cdot \overline{y} + y \cdot z.$$

a) Represent the values of F in a table;

b) Find the complete sum-of-products expansion of $F(x, y, z)$.

B) Write the dual of the expression

$$x \cdot y + \overline{(x + y) \cdot x} + \overline{y} = 1.$$

C) a) Use K-maps to minimize the Boolean function

$$F(x, y, z) = \overline{x}yz + xyz + x\overline{y}\,\overline{z} + x\overline{y}z.$$

b) Draw the logic gates (circuits) representing the minimized function $F(x, y, z)$ obtained at a).