PHYS 500
Final Exam
Thursday 5th January 2017

Instructor: Dr. V. Lempesis

Student Grade: ……./40

Please answer all questions on the booklet given. DO NOT write your answers on the questions sheet. You are allowed to use only your calculators. No tablets, laptops or mobiles are allowed during the exam.

1. In an experiment to measure the period of a pendulum we got the following recordings (in s): 3.2, 3.8, 3.5, 3.7, 3.6, 2.8. We have to apply the Chauvenet criterion **for one of them** to check if we are going to keep or drop it.

(a) For which of the results we have to apply the Chauvenet criterion?

(1 mark)

We are going to apply it for the sixth measurement 2.8

(b) Decide if you are going to keep it or to drop it.

(10 marks)

We apply the Chauvenet's criterion

$$\bar{T} = \sum_{i=1}^{6} T_i / 6 = \frac{3.2+3.8+3.5+3.7+3.6+2.8}{6} = 3.433333334\ s$$

Then we calculate the standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{6}(\bar{T}-T_i)^2}{N-1}} =$$

$$\sqrt{\frac{\left(3.2-3.43\right)^2+\left(3.8-3.43\right)^2+3.43^2+\left(3.7-3.43\right)^2+\left(3.6-3.43\right)^2+\left(2.8-3.43\right)^2}{6-1}} =$$

$$= 0.3723797345$$

Then we calculate the quantity μ:

$$\mu = \frac{\left|\bar{T}-T_6\right|}{\sigma} = \frac{\left|3.43-2.8\right|}{0.3723797345} = 1.700772828 \approx 1.7$$

Then we find the probability from the table $P(\leq \mu) = 0.0.9109$

So $P(\geq \mu) = 1 - P(\leq \mu) = 0.0891$

Then we find the quantity u:

$u = N \cdot P(\geq \mu) = 6 \cdot 0.0891 = 0.5346$ *this value is larger than 0.5 so the "bad value" 2.8 is accepted*

(d) Calculate the error of the average value

(2 mark)

$$\delta\overline{T} = \sqrt{\frac{\sum_{i=1}^{6}\left(T_i - \overline{T}\right)^2}{6 \cdot 5}} =$$

$$\sqrt{\frac{(3.2-3.43)^2 + (3.8-3.43)^2 + (3.5-3.43)^2 + (3.7-3.43)^2 + (3.6-3.43)^2 + (2.8-3.43)^2}{30}} = 0.1520233900s$$

(e) Quote the result for the average value.

(2 marks)

$$\overline{T} = (3.43 \pm 0.15)\ s$$

2. Measuring the length *L* and the width *d* of an orthogonal parallelogram we got the following recordings (in *mm*):

L	4.25	4.26	4.22	4.28	4.24
d	5.36	5.35	5.41	5.37	5.36

The errors for the above length and width measurements are $\delta L = \delta d = 0.04$ mm. Calculate the area of the parallelogram and its error with **two different ways**:

a) From L_i, d_i calculate the average values $\overline{L}$, $\overline{d}$ and their errors $\delta\overline{L}$ and $\delta\overline{d}$, and find S and δS.

(5 marks)

$$\bar{L} = \sum_{i=1}^{5} L_i / 5 = \frac{4.25 + 4.26 + 4.22 + 4.28 + 4.24}{5} = 4.25 \text{ mm}$$

$$\delta \bar{L} = \sqrt{\frac{\sum_{i=1}^{5}\left(L_i - \bar{L}\right)^2}{5\cdot 4}} =$$

$$\sqrt{\frac{(4.25-4.25)^2+(4.26-4.25)^2+(4.22-4.25)^2+(4.28-4.25)^2+(4.24-4.25)^2}{20}} = 0.010 \text{ mm}$$

$\bar{L} = (4.250 \pm 0.010)$ mm.

$$\bar{d} = \sum_{i=1}^{5} d_i / 5 = \frac{5.36+5.35+5.41+5.37+5.36}{5} = 5.37 \text{ mm}$$

$$\delta \bar{d} = \sqrt{\frac{\sum_{i=1}^{5}\left(d_i - \bar{d}\right)^2}{5\cdot 4}} =$$

$$\sqrt{\frac{(5.36-5.37)^2+(5.35-5.37)^2+(5.41-5.37)^2+(5.37-5.37)^2+(5.36-5.37)^2}{20}} = 0.010 \text{ mm}$$

$\bar{d} = (5.370 \pm 0.010)$ mm

$S = \bar{L}\cdot\bar{d} = 4.25\cdot 5.37 = 22.82250000 \text{ mm}^2$

$$\delta S = \sqrt{\left(\frac{\partial S}{\partial \bar{L}}\delta\bar{L}\right)^2 + \left(\frac{\partial S}{\partial \bar{d}}\delta\bar{d}\right)^2} = \sqrt{\left(\bar{d}\delta\bar{L}\right)^2 + \left(\bar{L}\delta\bar{d}\right)^2} = 0.06848313661 \text{ mm}^2.$$

Thus

$S = (22.82 \pm 0.07) \text{ mm}^2$

b) From L_i, d_i calculate each S_i, and from these calculate $\bar{S}$ and $\delta\bar{S}$.

(5 marks)

L	4.25	4.26	4.22	4.28	4.24
d	5.36	5.35	5.41	5.37	5.36
S	22.78	22.7910	22.8302	22.9836	22.7264

$$\bar{S} = \sum_{i=1}^{5} s_i / 5 = \frac{22.78+22.7910+22.8302+22.9836+22.7264}{5} = 22.82224000 \text{ mm}^2$$

$$\delta\bar{s} = \sqrt{\frac{\sum_{i=1}^{5}\left(S_i - \bar{S}\right)^2}{5\cdot 4}} =$$

$$\sqrt{\frac{\left(22.78-22.8222\right)^2+\left(22.7910-22.8222\right)^2+\left(22.8302-22.8222\right)^2+\left(22.9836-22.8222\right)^2+\left(22.7264-22.8222\right)^2}{20}}$$

$= 0.04361529090$ mm

Thus

$$\bar{S} = \left(22.82 \pm 0.04\right)\ \text{mm}^2.$$

c) Since we used **two different methods** calculate the final value of the area and its error, which we must quote.

(5 marks)

We calculate first the quantities $w_i = 1/\left(\delta S_i\right)^2$.

$w_1 = 1/\left(\delta S_1\right)^2 = 1/0.07^2 = 204.081$, $w_2 = 1/\left(\delta S_2\right)^2 = 1/0.04^2 = 625$

Also $\sum_{i=1}^{2} w_i = 829.08$

So the value for the velocity will be given by

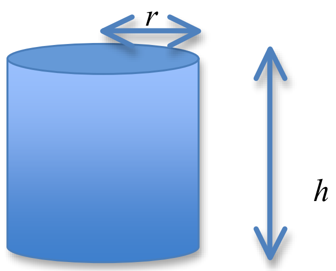
$$S = \frac{\sum_{i=1}^{2} S_i w_i}{\sum_{i=1}^{2} w_i} = \frac{S_1 w_1 + S_2 w_2}{\sum_{i=1}^{2} w_i} = 22.82\ \text{mm}^2$$

$$\delta S = \sqrt{\frac{1}{\sum_{i=1}^{2} w_i}} = 0.03472972\ mm^2$$

So

$$S = \left(22.82 \pm 0.03\right) m/s.$$

3. For the cylinder below we are given that its height is $h = (10.0 \pm 0.3)\ cm$, its radius is $r = (3.4 \pm 0.3)\ cm$ and its mass $M = (56.3 \pm 0.4)\, gr$.



a) Calculate the density of the cylinder.

(2 marks)

$$d = m / V = m / \left(\pi r^2 h\right) = 0.1551032553 \text{ g/cm}^3$$

b) Calculate the error in the density and round it to correct number of significant digits

(5 marks)

$$\delta d = \sqrt{\left(\frac{\partial d}{\partial r}\delta r\right)^2 + \left(\frac{\partial d}{\partial h}\delta h\right)^2 + \left(\frac{\partial d}{\partial M}\delta M\right)^2} =$$

$$\sqrt{\left(-\frac{2M}{\pi r^3 h}\delta r\right)^2 + \left(-\frac{M}{\pi r^2 h^2}\delta h\right)^2 + \left(\frac{1}{\pi r^2 h}\delta M\right)^2} = 0.0277857 \text{ g/cm}^2$$

$$\delta d = 0.028 \text{ g/cm}^2$$

c) Quote the result for the density

(2 marks)

$$d = (0.155 \pm 0.028) \text{ g/cm}^3$$

d) Calculate the relative error (in %) of the density.

(1 mark)

$$\eta = \frac{\delta d}{d} \times 100\% = \frac{0.028}{0.155} \times 100\% = 18.06\%$$

MATHEMATICAL FORMULAS

- Error in average value: $\delta x = \sqrt{\dfrac{\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2}{N(N-1)}}$

- Measurements with different method:

$$x = \left(\sum_{i=1}^{N} w_i x_i\right) / \left(\sum_{i=1}^{N} w_i\right), \quad w_i = 1/\left(\delta x_i\right)^2, \ \delta x = \sqrt{1/\left(\sum_{i=1}^{N} w_i\right)}$$

- Chauvenet's criterion: Standard deviation $\sigma = \sqrt{\frac{\sum_{i=1}^{N}\left(\bar{x} - x_i\right)^2}{N-1}}$, $\mu = \frac{\left|\bar{x} - x_j\right|}{\sigma}$,

  $P(\le \mu)$ from table, $P(\ge \mu) = 1 - P(\le \mu)$, $u = NP(\ge \mu)$
- Cylinder volume $V = \pi r^2 h$.

μ	**0,00**	**0,01**	**0,02**	**0,03**	**0,04**	**0,05**	**0,06**	**0,07**	**0,08**	**0,09**
0,0	0,00	0,80	1,60	2,39	3,19	3,99	4,78	5,58	6,38	7,17
0,1	7,97	8,76	9,55	10,34	11,13	11,92	12,71	13,50	14,28	15,07
0,2	15,85	16,63	17,41	18,19	18,97	19,74	20,51	21,28	22,05	22,82
0,3	23,58	23,34	25,10	25,86	26,61	27,37	28,12	28,86	29,61	30,35
0,4	31,08	31,82	32,55	33,28	34,01	34,73	35,45	36,16	36,88	37,59
0,5	38,29	38,99	39,69	40,39	41,08	41,77	42,45	43,13	43,81	44,48
0,6	45,15	45,81	46,47	47,13	47,78	48,43	49,07	49,71	50,35	50,98
0,7	51,61	52,23	52,85	53,46	54,07	54,67	55,27	55,87	56,46	57,05
0,8	57,63	58,21	58,78	59,35	59,91	60,47	61,02	61,57	62,11	62,65
0,9	63,19	63,72	64,24	64,76	65,28	65,79	66,29	66,80	67,29	67,78
1,0	68,27	68,57	69,23	69,70	70,17	70,63	71,09	71,54	71,99	72,43
1,1	72,87	73,30	73,73	74,15	74,57	74,99	75,40	75,80	76,20	76,60
1,2	76,99	77,37	77,75	78,13	78,50	78,87	79,23	79,59	79,95	80,29
1,3	80,64	80,98	81,32	81,65	81,98	82,30	82,62	82,93	83,24	83,55
1,4	83,85	84,15	84,44	84,73	85,01	85,29	85,57	85,84	86,11	86,38
1,5	86,64	86,90	87,15	87,40	87,64	87,89	88,12	88,36	88,59	88,82
1,6	89,04	89,26	89,48	89,69	89,90	90,11	90,31	90,51	90,70	90,90
1,7	91,09	91,27	91,46	91,64	91,81	91,99	92,16	92,33	92,49	92,65
1,8	92,81	92,97	93,12	93,28	93,42	93,57	93,71	93,85	93,99	94,12
1,9	94,26	94,39	94,51	94,64	94,76	94,88	95,00	95,12	95,23	95,34
2,0	95,45	95,56	95,66	95,76	95,86	95,96	96,06	96,15	96,25	96,34
2,1	96,43	96,51	96,60	96,68	96,76	96,84	96,92	97,00	97,07	97,15
2,2	97,22	97,29	97,36	97,43	97,49	97,56	97,62	97,68	97,74	97,80
2,3	97,86	97,91	97,97	98,02	98,07	98,12	98,17	98,22	98,27	98,32
2,4	98,36	98,40	98,45	98,49	98,53	98,57	98,61	98,65	98,69	98,72
2,5	98,76	98,79	98,83	98,86	98,89	98,92	98,95	98,98	99,01	99,04
2,6	99,07	99,09	99,12	99,15	99,17	99,20	99,22	99,24	99,26	99,29
2,7	99,31	99,33	99,35	99,37	99,39	99,40	99,42	99,44	99,46	99,47
2,8	99,49	99,50	99,52	99,53	99,55	99,56	99,58	99,59	99,60	99,61
2,9	99,63	99,64	99,65	99,66	99,67	99,68	99,69	99,70	99,71	99,72
3,0	99,73									
3,5	99,95									
4,0	99,994									
4,5	99,9993									
5,0	99,99994									

The values in the table are given in %