**Final Exam 425 Math**

# Question1

**Solve:**

**a)**$(D\_{x}^{2}-D\_{x}D\_{y}+D\_{y}-1)u=cos\left(x+2y\right)+xy+1.$

**b)**$(3D\_{x}D\_{y}-2\left(D\_{y})^{2}-D\_{y}\right)u=sin\left(2x+3y\right).$

# Question2

**a)Solve the Cauchy problem:**

$yu\_{xx}+\left(x+y\right)u\_{xy}+xu\_{yy}=xy,$

$u\left(0,y\right)=y ,u\_{x}\left(0,y\right)=1,y>0.$

**b) Find the integral surface of:**

$$x^{2}u\_{x}+y^{2}u\_{y}+u^{2}=0,which passes through the hyperbola :$$

$$ xy=x+y,u=1.$$

# Question3

**a)Does the following problem has a unique solution? Justify your answer.**

$Δu=0 in C:x^{2}+y^{2}=1,$

$u\_{n}=1 on ∂C.$

**b) Use D'Alembert solution to solve the problem:**

$u\_{tt}-4u\_{xx}=0, -\infty <x<\infty ,t\geq 0$

$u\left(x,0\right)=e^{-x},u\_{t}(x,0)=0,-\infty <x<\infty .$

## Question4

**Solve the Heat equation :** $u\_{t}=4u\_{xx}$

 **Subject to :** $u\_{x}\left(0,t\right)=u\_{x}\left(L,t\right)=0.$

$u\left(x,0\right)=6+4cos(\frac{3πx}{L})$ **.**

## Question5

**Find the potential** $φ $**inside the cylinder :**

$0\leq r\leq a ,0\leq θ\leq 2π,0\leq z\leq h,$

$$if the potential on the top z=h and on the lateral surface r=a$$

$$is held at zero,while on the base z=0 the potential $$

$φ\left(r,θ,0\right)=f\left(r,θ\right)$**, where f is acontinuous function such that** $f\left(a,θ\right)=0,$

$$0\leq θ\leq 2π.Consider the particular case f\left(r,θ\right)=cosθ.$$

 **Good Luck**

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