College of Science
Department of Mathematics

Final Examination

Sunday, January 7, 2018	Math 473	Academic year 1438-39H
8:00 - 11:00 am	Introduction to Differential Geometry	First Semester

Student's Name	
ID number	
Section No.	
Classroom No.	
Teacher's Name	**Dr Nasser Bin Turki**
Roll Number	

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Instructions:

- Your student identity card must be visible on your desk during the entire examination.
- Full marks can be obtained for complete answers to all FIVE questions.

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1. Let $\alpha : I \mapsto \mathbb{R}^3$ be a unit speed space curve. Define the unit tangent T and the curvature κ. Assuming that the curvature $\kappa(t) \neq 0$ for all $t \in I$, define the principal normal N, the binormal B and the torsion τ. Prove that $(\alpha' \times \alpha'') \bullet \alpha''' = \kappa^2 \tau$.

[8 marks]

2. Let a, b and c be real numbers, $a > 0$. Let $\alpha : \mathbb{R} \mapsto \mathbb{R}^3$ be the helix

$$\alpha(t) = (a\cos(bt), a\sin(bt), ct).$$

(a) Find the condition on a, b, c for the helix to be unit speed.

(b) Assume that the condition for the helix to be unit speed is satisfied.

(i) Compute the unit tangent T.

(ii) Compute the curvature κ.

(iii) Find the Serret-Frenet basis (Frame) of α.

[8 marks]

3. Let $X(u,v) = (u+v, u-v, uv)$.

(a) Show that X defines a regular surface patch.

(b) Calculate the coefficients E, F, G of the first fundamental form for this surface.

(c) Write down an integral which gives the length of the curve $\gamma_1(t) = X(t,1)$ on this surface from $t=1$ to $t=2$. You do not need to evaluate this integral.

(d) Calculate the cosine of the angle between the coordinate curves

$$\alpha_1(t) = X(t,1) \quad \text{and} \quad \alpha_2(t) = X(1,t)$$

on the surface at the point $X(1,1) = (2,0,1)$, where the curves α_1 and α_2 meet.

(e) Is the surface patch X conformal. **Why**.

[10 marks]

4. Let $X : U \subset \mathbb{R}^2 \mapsto \mathbb{R}^3$ be a regular surface patch given by $X(u,v) = (\cos u, \sin u, v)$.

(a) Compute the first fundamental form of the surface X. Compute the second fundamental form of the surface X. Compute the principal curvatures of this surface. Is there umbilic point(s).

(b) Compute the Gauss curvature and the mean curvature of this surface. Determine whether the surface is hyperbolic, parabolic or elliptic.

[10 marks]

5. Let $X : U \mapsto \mathbb{R}^3$ be a surface patch given by $X(u,v) = (u, v, \pi)$.

(a) Compute the Christoel symbols Γ^k_{ij} of X.

(b) Compute the coefficients β^j_i of X.

(c) Compute the Gauss-Weingarten equations of the surface X.

[4 marks]