

NAME:

Group Number/Instructor's Name:

ID:

Question	Grade
I	
II	
III	
IV	
Total	

Question	1	2	3	4	5	6	7	8
Answer								

I) Choose the correct answer (write it on the table above) [8 marks]:

1) If  $\ln(e^{4x}) - \ln(e) = 3$ , then  $x$  equals

(A) 0	(B) 1	(C) 2	(D) None
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2) If  $y = \sinh^{-1}(\tanh x)$ , then  $\frac{dy}{dx}$  equals

(A) $\frac{\operatorname{sech}^2 x}{1 + \tanh^2 x}$	(B) $\frac{\operatorname{sech}^2 x}{\sqrt{1 + \tanh^2 x}}$	(C) $\frac{\operatorname{sech}^2 x}{\sqrt{\tanh^2 x - 1}}$	(D) None
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3) If  $f(x) = \sinh(x) + \cosh(x)$ , then  $f(0)$  equals

(A) 0	(B) 1	(C) $e$	(D) None
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4) The graph of  $r = \sin 3\theta$  is

(A) a circle	(B) a rose with 3 leaves	(C) a rose with 6 leaves	(D) None
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5) The partial fraction decomposition of  $\frac{x^2 - 5}{x^4 + x^3 + x^2}$  is

(A) $\frac{A}{x^2} + \frac{B}{x^2 + x + 1}$	(B) $\frac{Ax + B}{x^2} + \frac{Cx + D}{x^2 + x + 1}$	(C) $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + x + 1}$	(D) None
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6) The rectangular coordinates  $(x, y)$  corresponding to the polar coordinates  $(r, \theta) = (3, \pi)$  are

(A) $(-3, 0)$	(B) $(3, 0)$	(C) $(0, -3)$	(D) None
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7) If  $f(x) = 10^x$ , then  $f'(x)$  equals

(A) $10^x$	(B) $10^x \ln 10$	(C) $\frac{10^x}{\ln 10}$	(D) None
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8)  $\frac{d}{dx} \int_0^1 \frac{x^3 + x^2}{5x^{3/2}} dx$  equals

(A) 1	(B) 2	(C) 0	(D) None
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II) Evaluate the following integrals [20 marks]:

1)  $\int x \cosh x dx$

2)  $\int \sin^4 x \cos^3 x dx$

3)  $\int \frac{x^3 + 2x}{x^2 - x - 2} dx$

4)  $\int \frac{e^{2x}}{\sqrt[3]{1 + e^x}} dx$

$$5) \int_0^3 \sqrt{3x}(\sqrt{x} + \sqrt{3})dx$$

$$6) \int \sqrt{x} \ln x dx$$

$$7) \int \frac{x^2}{(1-9x^2)^{3/2}} dx$$

$$8) \int \frac{1}{x^2 + 2x + 5} dx$$

III) A) Determine whether the improper integrals converge and, if so, find their value [5 marks]:

1)  $\int_{-8}^1 \frac{1}{\sqrt[3]{x}} dx$

2)  $\int_4^{\infty} \frac{1}{x-4} dx.$

B) Sketch the graph of the function  $f(x) = e^x$  and compute the area of the region that lies under the graph of  $y = e^x$ , over the  $x$ -axis and to the left of  $x = 0$  [3 marks].

IV) A) Sketch the graph of the curve with polar equation  $r = 3 - 3 \sin \theta$ , for  $0 \leq \theta \leq 2\pi$   
[3 marks].

B) Find the area of the region enclosed by the graph of  $r = 3 - 3 \sin \theta$ , for  $0 \leq \theta \leq 2\pi$   
[3 marks].

C) Find a polar equation that has the same graph as  $xy = 4$  [2 marks].

## Formulas

- $\int \frac{1}{\sqrt{a^2 + u^2}} du = \sinh^{-1} \frac{u}{a} + c, a > 0;$
- $\int \frac{1}{a^2 - u^2} du = \frac{1}{a} \tanh^{-1} \frac{u}{a} + c, |u| < a;$
- $\int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1} \frac{u}{a} + c, a > 0;$
- $\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \tan^{-1} \frac{u}{a} + c, a > 0.$

Scrap paper