King Saud University
Department of Mathematics

106
Final Exam, December 2015

NAME:

Group Number/Instructor's Name:

ID:

Question	Grade
I	
II	
III	
IV	
V	
Total	

Question	1	2	3	4	5	6	7	8	9	10
Answer										

I) Choose the correct answer (write it on the table above):

1) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{i}{n^2}$ equals

(A) $\frac{1}{2}$	(B) 1	(C) 0	(D) None

2) $\int\limits_0^1 \ln(e^{2x})dx$ equals

(A) 0	(B) e	(C) 1	(D) None

3) If the polar coordinates of a point are $(r, \theta) = \left(-3, \frac{3\pi}{2}\right)$, then its rectangular coordinates are

(A) $(x, y) = (-3, 0)$	(B) $(x, y) = (0, 3)$	(C) $(x, y) = (0, -3)$	(D) None

4) If $f'(x) = 2x$ and $f(0) = 2$, then

(A) $f(x) = x^2$	(B) $f(x) = x^2 + 2$	(C) $f(x) = 2x^2$	(D) None

5) If $\int\limits_1^3 f(x)dx = 3$ and $\int\limits_1^7 f(x)dx = 7$, then

(A) $\int\limits_1^4 f(x)dx = 4$	(B) $\int\limits_1^{10} f(x)dx = 10$	(C) $\int\limits_3^7 f(x)dx = 4$	(D) None

6) The number c satisfying the Mean Value Theorem for $\int\limits_0^2 x dx$ is

(A) $c = 1$	(B) $c = 0$	(C) $c = -1$	(D) None

7) If $x = \log_3 9$, then

(A) $x = 6$	(B) $x = 3$	(C) $x = 2$	(D) None

8) If $\sinh x = 3$, then

(A) $\cosh x = \sqrt{10}$	(B) $\cosh x = -\sqrt{10}$	(C) $\cosh x = 2\sqrt{2}$	(D) None

9) $\int \tan(2x)dx$ equals

(A) $\frac{1}{2}\ln\|\cos(2x)\|+c$	(B) $-\frac{1}{2}\ln\|\cos(2x)\|+c$	(C) $\ln\|\cos(2x)\|+c$	(D) None

10) The Simpson's rule for approximating $\int_1^3 \frac{1}{x}dx$, for $n = 4$, is

(A) $\frac{1}{6}\left(1+\frac{8}{3}+1+\frac{8}{5}+\frac{1}{3}\right)$	(B) $\frac{1}{4}\left(1+\frac{4}{3}+1+\frac{4}{5}+\frac{1}{3}\right)$	(C) $\frac{1}{2}\left(\frac{4}{5}+1+\frac{4}{7}+\frac{4}{9}+\frac{4}{11}\right)$	(D) None

II) A) Compute the following integrals:

1) $\int \left(\frac{3}{1+x^2} + 2e^{-x} + \frac{1}{x}\right) dx$

2) $\int \sin^{-1} x dx$

3) $\int x\sqrt{x^2+1} dx$

B) If $F(x) = \int\limits_0^{x^2} \tan(t^2) dt$, find $F'(x)$.

III) A) Compute the following integrals:

1) $\displaystyle\int \frac{2x}{x^2-6x+9}dx$

2) $\displaystyle\int \tan^{3/2} x \sec^4 x dx.$

B) Determine whether the improper integrals converge or diverge:

1) $\int\limits_{0}^{1} \frac{x}{x^2 - 1} dx$

2) $\int\limits_{2}^{\infty} \frac{1}{\sqrt{x^3}} dx$

C) Solve for x the equation $\cosh(\ln x) = 1$.

IV) A) Find the arc length of the portion of the curve $y = \dfrac{2}{3}x\sqrt{x}$, for $0 \leq x \leq 3$.

B) Find the area of the region enclosed by the graphs of $y = x^2$, $y = -x + 2$ and the x-axis.

C) Find the volume of the solid resulting from revolving about the x-axis the region enclosed by the graph of $y = \sqrt{x}$ and the x-axis, on the interval $[0, 1]$.

D) Find the volume of the solid resulting from revolving about the y-axis the region enclosed by the graphs of $y = x^2 + 2$, $y = 6$ and $x = 0$.

V) A) Find the equation, in rectangular coordinates, of the curve whose polar equation is

$$r\sin\theta = \ln r + \ln\cos\theta.$$

B) Compute the area of the region enclosed by the graph of $r = 2 - 2\sin\theta$, for $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

C) Sketch the graph of the curve whose parametric equations are $x = t$, $y = t^2 + 1$, for $0 \le t \le 3$. Then, find the equation of the curve, in rectangular coordinates.

- $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$
- $\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$, $x \geq 1$
- $\tanh^{-1} x = \dfrac{1}{2} \ln \dfrac{1 + x}{1 - x}$, $|x| < 1$
- $\operatorname{sech}^{-1} x = \ln \dfrac{1 + \sqrt{1 - x^2}}{x}$, $0 < x \leq 1$
- $\dfrac{d}{dx}(\sinh^{-1} x) = \dfrac{1}{\sqrt{x^2 + 1}}$
- $\dfrac{d}{dx}(\cosh^{-1} x) = \dfrac{1}{\sqrt{x^2 - 1}}$, $x > 1$
- $\dfrac{d}{dx}(\tanh^{-1} x) = \dfrac{1}{1 - x^2}$, $|x| < 1$
- $\dfrac{d}{dx}(\operatorname{sech}^{-1} x) = -\dfrac{1}{x\sqrt{1 - x^2}}$, $0 < x < 1$
- $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1 - x^2}}$, $|x| < 1$
- $\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1 - x^2}}$, $|x| < 1$
- $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1 + x^2}$
- $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{|x|\sqrt{x^2 - 1}}$, $|x| > 1$