King Saud University
Mathematics Department
Math 244
Final Exam

Name:
ID:
Section:
Teacher:

Quiztion	I	II	III	IV	V	VI	Total
mark							

I. Determine if the statement is always true or sometimes false, and justify your answer with a logical argument or a counter example.

1) The set $\{1, x, e^x\}$ is linearly independent. ( )

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2) The reflection operator about the x-axis in R^2 is one-to-one. ( )

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3) Whenever 4 is an eigenvalue of a matrix A then 12 is an eigenvalue of A^3. ( )

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4) $V = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, a, b \in R \right\}$ with standard addition and scalar multiplication of matrices, is a subspace of $M_{2\times2}$. ( )

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5) $W = \{(a,b); a, b \in R, and\ a^2 = b^2\}$ is a suspace of R^2. ( )

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II. Choose the correct answer:

1) If B is 5×7 and nullity$(B)=3$ then nullity of (B^t) is

a) 2 b) 5 c) 3 d) 1

2) If $v_1=(2,1)$, $v_2=(8,4)$ then the set $\{v_1,v_2\}$

a) is a basis of R^2. b) spans R^2.

c) linearly dependent. d) linearly independent.

3) The vector (a,a,b) is a linear combination of the vectors $(0,1,-1)$, $(1,-1,0)$ if the relation between a and b is

a) $a=2b$ b) $b=2a$ c) $a=-2b$ d) $b=-2a$

4) If $A=\begin{bmatrix}1 & -1 & 3\\ 2 & -3 & 6\\ -1 & 4 & -2\end{bmatrix}$, then $A(adj(A))=$

a) $-4I$ b) $2I$ c) $-2I$ d) $-I$

5) If $W=span\{(1,-1,0,1),(-1,1,1,0),(2,-2,1,3)\}$ then

a) $\dim W=1$ b) $\dim W=3$ c) $\dim W=2$ d) $\dim W=4$

6) If A is 4×6 then

a) The column vectors are linearly dependent.
b) The column vectors are linearly independent.
c) dim (column space) = 6.
d) column space(A) = row space (A).

7) The values of λ which make the vectors $\{(1,-1,-\lambda),(1,2,-\lambda),(\lambda,0,1-2\lambda)\}$ linearly independent are

a) R b) $R\setminus\{1\}$ c) $R\setminus\{-1\}$ d) $R\setminus\{1,-1\}$

8) If u,v are vectors in a vector space V such that $\|2u+3v\|=\|2u-3v\|$ then

a) $u=\frac{3}{2}v$ b) $u\cdot v=0$ c) $u=\frac{-3}{2}v$ d) $u=v$

Question number	1	2	3	4	5	6	7	8
Answer								

III.

1) **Solve the following linear system by <u>Guassion elimination</u>**

2)
$$\begin{aligned} -x_1 &+ x_2 &+ x_3 &+ 3x_4 &= 1 \\ 3x_1 &- 2x_2 &- 3x_3 &- 2x_4 &= -4 \\ 2x_1 &+ x_2 &- 2x_3 &+ x_4 &= -3 \end{aligned}$$

3) Consider the system
$$\begin{aligned} x &+ a^2 y &= 2a \\ x &+ y &= -2 \end{aligned}$$

Find values of a so that :

a) The system has one solution.
b) The system has infinitely many solutions.
c) The system is inconsistent.

IV.

1) Let $T : R^2 \to R^3$ be defined by $T(x, y) = (x - 2y, 2x + y, 3x)$

   a) Show that T is a linear transformation.

   b) Find the standard matrix of T .

2) If $T : R^2 \to R^2$ is a linear transformation such that $T(1,-2) = (-1,1)$, and $T(-2,3) = (0,-2)$. Then find $T(x, y)$.

V.

1) Find a subset of the vectors that forms a basis for the space spanned by these vectors, then express each vector that is not in the basis as a linear combination of the basis vectors.

$v_1 = (1,-1,5,2)$ $v_2 = (-2,3,1,0)$ $v_3 = (4,-5,9,4)$ $v_4 = (0,4,2,-3)$ $v_5 = (-7,18,2,-8)$

2) If $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$, then

a) Find the rank and nullity of A.
b) Find a basis of the nullspace of A.

VI. Let $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$

a) Show that the distinct eigenvalues of **A** are **2,4.**
b) Find,if possible, a matrix **P** which diagonalizes **A**, and write it's **diagonal form**.