King Saud University
Faculty of Sciences
Department of Mathematics

Final Examination	Math 132	Semester I	1439-1440	Time: 2H

Exercise 1 : (4+5+4+5)

1. Without using truth tables, prove the following logical equivalence:
$$\neg[\neg p \wedge (q \rightarrow p)] \equiv p \vee q$$
2. Consider the sequence $\{a_n\}_{n=0}^{\infty}$ defined as follows:
$a_0 = 1, a_1 = 2$, and $a_{n+1} = 5a_n - 6a_{n-1}; \forall n \geq 1$.
Use mathematical induction to prove that $a_n = 2^n$, for each integer n, with $n \geq 0$.
3. Consider the partial ordering P on the set $A := \{a, b, c, d, e\}$ defined by:
$P = \{(a, a), (b, b), (c, c), (d, d), (e, e), (a, b), (c, b), (c, d), (d, e), (c, e)\}$.
   (a) Draw the Hasse diagram of P.
   (b) Is P a total ordering?
4. Let E be the relation defined on the set of real numbers as follows:
$$(a \; E \; b) \text{ if and only if } (a - b) \text{ is an integer.}$$
   (a) Prove that E is an equivalence relation.
   (b) What is the equivalence class [1], of the element 1 for this equivalence relation?
   (c) Is $\frac{3}{2} \in [\frac{2}{3}]$?

Exercise 2 : (8+3)

1. Consider the sets $A := \{a, b, c, d, e\}$ and $B := \{1, 2, 3, 4\}$, and the function $f : A \longrightarrow B$ defined by: $f(a) = 1, f(b) = f(c) = 2$, and $f(d) = f(e) = 3$.
   (a) Find the image of each of the sets $\{a, b, c\}, \{d, e\}$ and $\{b, c, d, e\}$.
   (b) Find the inverse image of each of the sets $\{1, 2, 3\}, \{3\}$ and $\{4\}$.
   (c) For the function f, determine whether it is one-to-one, and whether it is onto B. (Justify your answer).
2. Consider a nonempty set X, and let $f : X \longrightarrow X$ be a function.
Prove that if the composite $f \circ f$ is a bijection, then f is a bijection.

Exercise 3 : (3+8)

1. Give four infinite sets A, B, C and D such that: $\overline{\overline{A}} < \overline{\overline{B}} < \overline{\overline{C}} < \overline{\overline{D}}$. (Justify your answer).
2. Determine whether each of the following statements is true or false. (Justify your answer).
(i) The set $\{\frac{3}{5^k}; k \quad$ is an integer with $k \geq 1\}$ is denumerable.
(ii) If A is an infinite set, then its power set $\mathcal{P}(A)$ is an uncountable set.
(iii) Given two sets A and B, if $\overline{\overline{A}} \leq \overline{\overline{B}}$, then $A \subseteq B$.
(iv) If B is a finite subset of a denumerable set A, then the set $A - B$ is denumerable.