King Saud University
Department of Mathematics
First Semester 1436-1437 H

MATH 352 (Numerical Analysis)
Final Exam
Duration: 3 Hours

Student's Name	Student's ID	Group No.

Question No.	I	II	III	IV	V	VI	Total
Mark							

[I] Determine whether the following is **True** or **False**. **Justify** your answer. [6 *Points*]

(1) The number of iterations needed to solve $x^3 - 2^x$ on $[0, 2]$ with accuracy 10^{-3} by the Bisection method **is** 6. ( )

(2) The sequence $p_n = p_{n-1} - \frac{p_{n-1}^3 - 5}{8}$ converges to $\sqrt[3]{5}$ **faster** than the sequence $p_n = \sqrt{\frac{5}{p_{n-1}}}$. ( )

(3) If $f(x) = x^4 - 4x + 3$ then the Newton's Method for finding the root $p = 1$ of f converges **quadratically**. ( )

OVER

(4) If $A = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 5 & -2 \\ 1 & 0 & 4 \end{bmatrix}$ then the Jacobi Method for solving $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ is **convergent** for any initial vector $\mathbf{x}^{(0)}$.

( )

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(5) The matrix $C = \begin{bmatrix} 1.001 & 3 \\ 1 & 3 \end{bmatrix}$ is **ill-conditioned**. ( )

[II] Choose the correct answer. [9 *Points*]

(1) The data

x	-2	-1	0	1	2
f(x)	-1	3	1	-1	3

has the following **interpolation** polynomial

(a) $p(x) = 3-2(x+1)+x(x^2-1)$ **(b)** $p(x) = -1+4(x+2)-3(x+2)(x+1)$ **(c)** $\frac{-x(x^2-1)(x-2)}{24} + \frac{(x^2-4)(x-1)}{2}$

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(2) If $f(1) = 1$, $f(1.2) = 1.2625$ and $f(1.4) = 1.6595$, then the **Backward-Difference** formula to determine $f'(1.2)$ gives

(a) $f'(1.2) = 1.637$ **(b)** $f'(1.2) = 1.3125$ **(c)** $f'(1.2) = 1.985$

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(3) If the quadrature formula $\int_0^2 f(x)dx = c_0 f(0) + c_1 f(1) + c_2 f(2)$ is **exact** for all polynomials of degree less than or equal to 2, then

(a) $c_0 = c_1 = c_2 = \frac{1}{3}$ **(b)** $c_0 = c_2 = \frac{1}{3}$, $c_1 = \frac{4}{3}$ **(c)** $c_0 = c_1 = \frac{1}{3}$, $c_2 = \frac{4}{3}$

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(4) Solving the system

$$\begin{aligned} 2.53x_1 - 10.1x_2 + 11.7x_3 &= -99.2 \\ -2.53x_1 + 10.1x_2 - 5.83x_3 &= 100 \\ 5.10x_1 - 11.8x_2 + 17.5x_3 &= -116 \end{aligned}$$

by Gaussian elimination with **scaled partial pivoting**, the first row interchange needed is

(a) $R_1 \longleftrightarrow R_2$ **(b)** $R_1 \longleftrightarrow R_3$ **(c)** $R_2 \longleftrightarrow R_3$

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(5) If $T = \begin{bmatrix} 3 & -2 & 5 \\ 1 & 0 & 7 \\ 0 & 0 & 2 \end{bmatrix}$, then $\rho(T)$ equals

(a) 1 **(b)** 2 **(c)** 3

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(6) If $M = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, then $\|M\|_2$ **equals**

(a) 3 **(b)** 9 **(c)** $7 + \sqrt{40}$

OVER

[III] [6 *Points*]

(a) **Determine** the value of h that will ensure an approximation error of less than 10^{-4} when approximating $\int_0^\pi \cos x^2 dx$ by the Composite Trapezoidal Rule.

(b) **Approximate** $\int_0^\pi \cos x^2 dx$ using the Composite Midpoint Rule with $n = 6$.

(c) **How accurate** is the approximation in (b)?

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[IV] [7 *Points*]

(i) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 7 \\ -1 & 2 & 5 \end{bmatrix}$.

(a) **Find** the permutation matrix P, a lower triangular matrix L with ones on its diagonal and an upper triangular matrix U so that $PA = LU$.

(b) Use (a) to **show** that A is invertible.

(ii) Let $B = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 5 \end{bmatrix}$.

(a) **Prove** that B is positive definite.

(b) **Factorize** B as LDL^T, where L is a lower triangular matrix and D is a diagonal matrix.

OVER

[V] Let $A = \begin{bmatrix} 0.04 & 51.2 \\ -6.2 & 8.1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 55.2 \\ -53.9 \end{bmatrix}$. [6 *Points*]

(a) Use Gaussian elimination with **partial pivoting** and 3-digit **chopping** arithmetic to approximate the solution of the system $A\mathbf{x} = \mathbf{b}$.

(b) **Compute** the residual vector $\mathbf{r} = \mathbf{b} - A\widetilde{\mathbf{x}}$, where $\widetilde{\mathbf{x}}$ is the approximation in (a).

(c) Use **one** iteration of the **iterative refinement** technique to improve $\widetilde{\mathbf{x}}$.

OVER

[VI] Consider the system [6 *Points*]

$$\begin{aligned} 2x_1 - x_2 + x_3 &= -1 \\ 2x_1 + 2x_2 + x_3 &= 4 \\ -x_1 - x_2 + 2x_3 &= 5 \end{aligned}$$

(a) Find the second iteration $\mathbf{x^{(2)}}$ of the **Gauss-Seidel** method to approximate the solution of the system using $\mathbf{x^{(0)}} = \mathbf{0}$.

(b) **Compute** $\|\mathbf{x^{(2)}} - \mathbf{x^{(1)}}\|_\infty$.

Good Luck