MATH 352 (Numerical Analysis)
Final Exam
Duration: 3 Hours

Student's Name	Student's ID	Group No.

Question No.	I	II	III	IV	V	VI	Total
Mark							

[**I**] **Determine** whether the following is **True** or **False**. **Justify** your answer. [6 *Points*]

(1) The **Bisection method** generates a sequence $\{p_n\}$ approximating p with **rate of convergence** $O(2^{-n})$. (   )

(2) If the Composite Midpoint Rule is used to compute $\int_0^2 2x^3 - 9 \quad dx$ with an error of at most 0.5×10^{-4}, **then the number of points** should be used **is** 13. (   )

OVER

(4) The sequence $\mathbf{x}^{(k)} = \left(e^{1-k}, 2 + \frac{3}{k}, \sin(\frac{1}{k})\right)^T$ is **convergent**.

( )

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(5) The matrix $C = \begin{bmatrix} 2 & 5 \\ 0 & 0.3 \end{bmatrix}$ is **convergent**. ( )

[II] Choose the correct answer. [9 *Points*]

(1) If the **Modified Newton's Method** is used to find the root of $f(x) = e^{2x} - 2x - 1$ with $p_0 = 1$, **then** p_1 **equals**

(a) -0.6713 **(b)** 0.6565 **(c)** *zero* **(d)** None of the previous

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(2) If $f(1.2) = 1.6402$, $f(1.3) = 1.7047$ and $f(1.4) = 1.7127$, then the **Forward-Difference** formula to determine $f'(1.3)$ gives

(a) $f'(1.3) = 0.08$ **(b)** $f'(1.3) = 0.645$ **(c)** $f'(1.3) = 0.725$ **(d)** None of the previous

OVER

(3) If $\frac{1}{2}$ is approximated by a **third Lagrange Polynomial** for $f(x) = 2^x$ through the points $-2, 0, 2, 4$, then the approximate value is

(a) 0.7656 **(b)** 0.4909 **(c)** 0.4417 **(d)** None of the previous

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(4) If Gaussian elimination with **scaled partial pivoting** is used in solving the system

$$\begin{aligned} 3.03x_1 - 12.1x_2 + 14x_3 &= -119 \\ -3.03x_1 + 12.1x_2 - 7x_3 &= 120 \\ 6.11x_1 - 14.2x_2 + 21x_3 &= -139 \end{aligned}$$

then **the first row interchange** needed is

(a) $R_1 \longleftrightarrow R_2$ **(b)** $R_1 \longleftrightarrow R_3$ **(c)** $R_2 \longleftrightarrow R_3$ **(d)** None of the previous

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(5) If $B = \begin{bmatrix} 5 & 1 & 3 \\ 0 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, then $\rho(B)$ equals

(a) 1 **(b)** 2 **(c)** 5 **(d)** None of the previous

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(6) If $M = \begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix}$, then $\|M\|_2$ **equals**

(a) 6 **(b)** $15 + \sqrt{125}$ **(c)** 26 **(d)** None of the previous

OVER

[III] [5 *Points*]

(a) **Approximate** $\int_{8.1}^{8.9} x \ln(x) \quad dx$ using the Composite Simpson Rule with $h = 0.2$.

(b) **Find a bound** for the error term of the approximation in (a).

OVER

[IV] Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \\ -3 & -4 & 14 \end{bmatrix}$. [6 *Points*]

(a) **Show** that A is positive definite.

(b) **Find** the LU factorization of A where L is a lower triangular matrix with ones on its diagonal and U is an upper triangular matrix.

(c) Use the LU factorization in (b) to **solve** the system $A\mathbf{x} = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$

OVER

[V] Consider the system [7 *Points*]

$$\begin{aligned} 4x_1 - x_2 - x_3 &= 3 \\ -2x_1 + 6x_2 + x_3 &= 9 \\ -x_1 + x_2 + 7x_3 &= -6 \end{aligned}$$

(a) **Show** that if the Gauss-Seidel method is applied to solve the system, it gives a sequence of vectors that **converges** to the unique solution of the system for any choice of $\mathbf{x^{(0)}}$.

(b) **Find** the **second** iteration $\mathbf{x^{(2)}}$ of the **Gauss-Seidel** method to approximate the solution of the system using $\mathbf{x^{(0)}} = \mathbf{0}$.

(c) **Compute** $\|\mathbf{x^{(2)}} - \mathbf{x^{(1)}}\|_2$.

OVER

[VI] Let $A = \begin{bmatrix} 0.03 & 58.9 \\ 5.31 & -6.10 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 59.2 \\ 47.0 \end{bmatrix}$. [7 *Points*]

(a) Use Gaussian elimination with **partial pivoting** and 3-digit **chopping** arithmetic to **approximate** the solution of the system $A\mathbf{x} = \mathbf{b}$.

(b) **Compute** the residual vector $\mathbf{r} = \mathbf{b} - A\widetilde{\mathbf{x}}$, where $\widetilde{\mathbf{x}}$ is the approximation in (a).

(c) Use **one** iteration of the **iterative refinement** technique to improve $\widetilde{\mathbf{x}}$.

(d) **Is** A well-conditioned? **Justify your answer**.

Good Luck