Final Dec19 Grading scheme

Question 1

a) $\int_0^3 (4x - x^2)dx = 3(4c - c^2) \Leftrightarrow c^2 - 4c + 3 = 0$ **(1.5)**

$$c = 1 \text{ or } c = 3 \textbf{ (0,5)}$$

b) $\int \frac{dx}{\sqrt{5^x - 16}} = \frac{2}{ln5} \int \frac{du}{u\sqrt{u^2 - 16}} = \frac{1}{2ln5} \sec^{-1}\left(\frac{5^{\frac{x}{2}}}{4}\right) + C$ **(1.5) + (0.5)**

c) $\int \frac{cotxdx}{\sqrt{9-(sinx)^4}} = \frac{1}{2} \int \frac{du}{u\sqrt{9-u^2}}$ $u = (sinx)^2$ **(2)**

$$= \frac{-1}{6} \text{sech}^{-1}\left(\frac{(sinx)^2}{3}\right) + C \quad \textbf{(1)}$$

Question 2

a) $\lim_{x\to 3} \frac{\ln(x-2)-x+3}{(x-3)\ln(x-2)} = \lim_{x\to 3} \frac{\frac{1}{x-2}-1}{\ln(x-2)+\frac{x-3}{x-2}} = \lim_{x\to 3} \frac{\frac{-1}{(x-2)^2}}{\frac{1}{x-2}+\frac{1}{(x-2)^2}}$

$$= -\frac{1}{2} \quad \textbf{(1) + (1) + (1)}$$

b) $\int x^2 \tan^{-1}(x)dx = \frac{x^3}{3}\tan^{-1}(x) - \frac{1}{3}\int \frac{x^3}{1+x^2}dx$

$$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{3}\int (x - \frac{x}{1+x^2})dx$$

$$= \frac{x^3}{3}\tan^{-1} x - \frac{1}{6}x^2 + \frac{1}{6}\ln(1 + x^2) + C$$

$$\textbf{(1) + (1) + (1)}$$

c) $\int (tanx)^4 (secx)^6 dx = \int u^4 (u^2 - 1)^2 du$

$$= \frac{(tanx)^9}{9} - \frac{2}{7}(tanx)^7 + \frac{1}{5}(tanx)^5 + C$$

$$\textbf{(1.5) + (1.5)}$$

Question 3

a) $\int \frac{x^2 dx}{(x^2+9)^{3/2}} = \int \frac{(tan\theta)^2}{\sec(\theta)} d\theta = \int (sec\theta - cos\theta) d\theta \quad x = 3tan\theta \quad (\mathbf{1.5})$

$$= \ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| - \frac{x}{\sqrt{x^2+9}} + C \quad (\mathbf{1.5})$$

b) $\frac{3x-2}{(x+2)(x^2+4)} = \frac{-1}{x+2} + \frac{x+1}{x^2+4} \quad (\mathbf{1.5})$

$$\int \frac{(3x-2)dx}{(x+2)(x^2+4)} = -\ln|x+2| + \frac{1}{2} ln(1+x^2) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C \quad (\mathbf{1.5})$$

d) $\int \frac{dx}{3-sinx+cosx} = \frac{1}{2}\int \frac{du}{2-u+u^2} = \frac{1}{2} \int \frac{du}{(u-\frac{1}{2})^2+\frac{7}{4}} \quad (\mathbf{2})$

$$= \frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{2u-1}{\sqrt{7}}\right) + C = \frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{2\tan\left(\frac{x}{2}\right)-1}{\sqrt{7}}\right) + C(\mathbf{1})$$

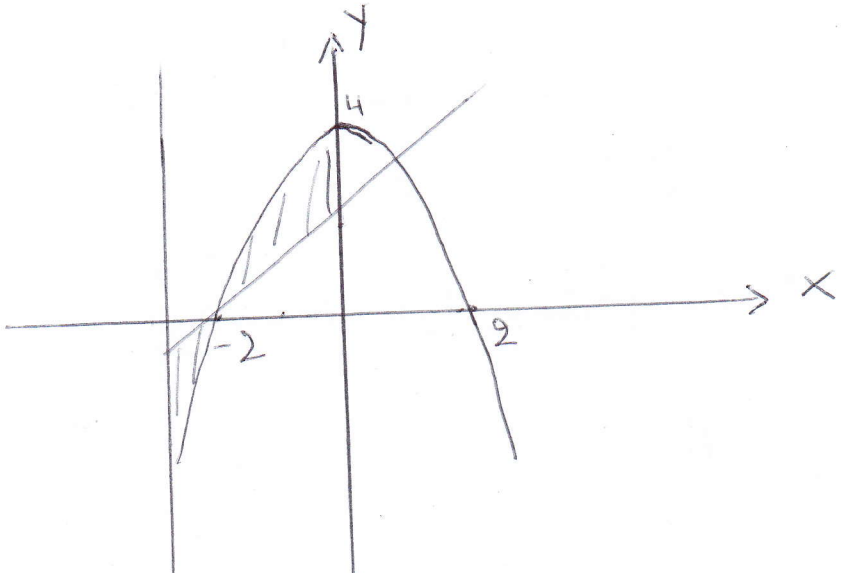
Question 4

a) $x + 2 = 4 - x^2 \Leftrightarrow x = -2 \; or \; x = 1$

$$A = \int_{-3}^{-2} x + 2 - (4 - x^2)dx + \int_{-2}^{0} 4 - x^2 - (x+2)dx \quad (\mathbf{1.5})$$

$$= \frac{11}{6} + \frac{10}{3} = \frac{31}{6} \quad (\mathbf{0.5})$$

graph (**1**)



b)$V = \int_{-1}^{1} 2\pi(3-x)(-x^2+1)dx = 8\pi$ **(2)**

c) $V = \int_{-1}^{1} \pi(9-(2+x^2)^2)dx$ **(1)**

Question5

a)$L = \int_0^{\pi} \sqrt{(\cos(\frac{\theta}{2}))^4 + (\cos(\frac{\theta}{2}))^2(\sin(\frac{\theta}{2}))^2}\, d\theta$ **(1.5)**

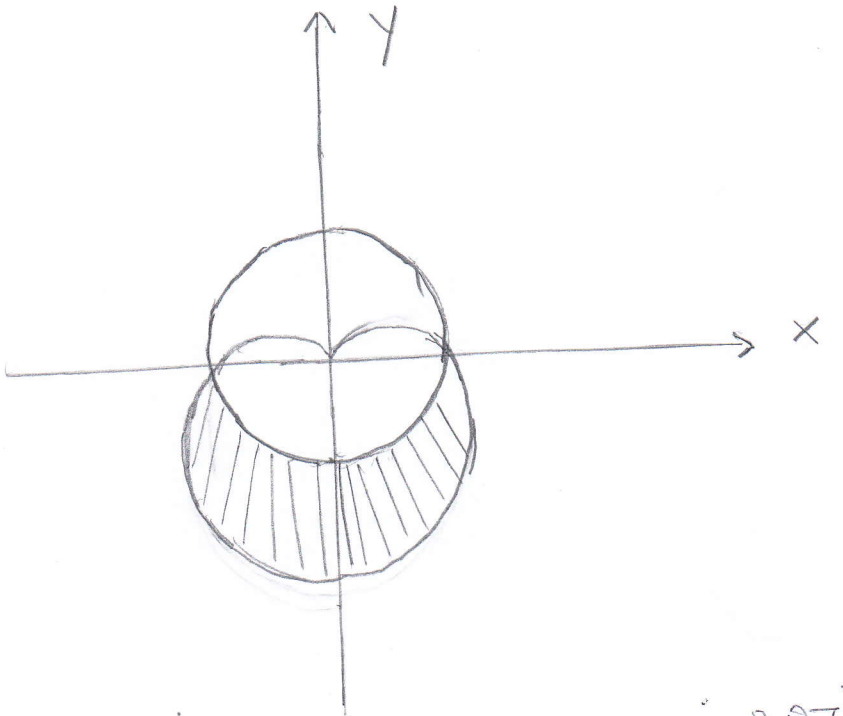
$= \int_0^{\pi} cos\left(\frac{\theta}{2}\right) d\theta = 2$ **(1.5)**

b)$1 - sin\theta = 1 \Rightarrow \theta = 0\ or\ \theta = \pi\ 0r\ \theta = 2\pi$

$A = \frac{1}{2}\int_{\pi}^{2\pi} [(1-sin\theta)^2 - 1]d\theta$) **(1)**

$= \frac{\pi}{4} + 2$ **(1)**

graph **(1)**



c) $S = 8\pi \int_0^{\pi/4} \cos^2\theta\, d\theta = 4\pi\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/4}$

$= 2\pi\left(\frac{\pi}{2}+1\right)$ (1.5) + (1.5)

$\simeq 0,6426$