King Saud University	College of Sciences	Department of Mathematics

Final Examination	Math 132	Semester 2(1445)	Time: 3H

Calculators are not allowed

The Examination contains 2 pages

Question 1: (8 marks)

1. Without using truth tables, prove the following logical equivalence:**(3 marks)**

$$[(\neg p \to q) \vee (\neg p \wedge \neg q)] \vee (\neg q \wedge p) \equiv T$$

2. Using a proof by contradiction, show that $\left(\dfrac{2+\sqrt{5}}{3}\right)$ is an irrational number.

   (Hint: use the fact that $\sqrt{5}$ is irrational).**(2 marks)**

3. Consider the sequence $\{a_n\}_{n=0}^{\infty}$ defined as follows:
   $\begin{cases} a_0 = a_1 = 1 \\ a_{n+1} = 4a_n - 4a_{n-1}, \text{ for all } n \geq 1. \end{cases}$
   Use mathematical induction to prove that $a_n = 2^n - n2^{n-1}$, $\forall n \geq 0$. **(3 marks)**

Question 2: (17 marks)

1. Let R be the relation from $A := \{1, 2, 3, 4\}$ to $B := \{0, 1, 2\}$, defined by

$$\text{for } a \in A,\ b \in B,\ [(aRb) \Leftrightarrow (a + b \leq 3)].$$

   (a) List all ordered pairs in R. **(2 marks)**

   (b) Represent R by a matrix. **(1 mark)**

   (c) Find the relations R^{-1}, $R \circ R^{-1}$ and $R^{-1} \circ R$. **(3 marks)**

2. Let E be the relation on the set of integers $\mathbb{Z}$ defined as follows:

$$\text{for } a, b \in \mathbb{Z},\ [(aEb) \Leftrightarrow (2|(a^2 + b^2)].$$

   (a) Show that E is an equivalence relation on $\mathbb{Z}$.**(3 marks)**

   (b) Decide whether $3 \in [2]$, justify your answer.**(1 mark)**

3. Let P be the relation defined on the set $C := \{a, b, c, d, e, f\}$ by
   $P = \{(a, a), (a, b), (a, c), (a, d), (a, e), (b, b), (b, e), (c, c), (c, f), (d, d), (a, f), (e, e), (f, f)\}$.

   (a) Draw the digraph of P. **(1 mark)**

   (b) Show that P is a partially ordering relation on C.**(3 marks)**

   (c) Is P a total ordering relation on C?**(1 mark)**

   (d) Draw the hasse diagram of P on the set C. **(2 marks)**

Question 3: (11 marks)

Consider the sets $X := \{a, b, c, d\}$ and $Y := \{0, 2, 4, 6\}$, and the function $f : X \longrightarrow Y$ defined by: $f(a) = f(d) = 4$, $f(b) = 6$ and $f(c) = 0$.

1. Find the image of each of the sets $\{a, b\}$ and $\{a, c, d\}$. **(2 marks)**
2. Find the inverse image of each of the sets $\{2\}$ and $\{0, 6\}$. **(2 marks)**
3. For the function f, determine whether it is one-to-one, and whether it is onto B. (Justify your answer). **(2 marks)**

Let g and h be the functions from the set of integers to the set of integers defined by $g(x) = x - 4$ and $h(x) = 2x^2 - 1$.

1. Prove that g is a one to one correspondence. **(2 marks)**
2. Find the g^{-1} the inverse function of g. **(1 mark)**
3. Find $g \circ h$ and $h \circ g$. **(2 marks)**

Question 4: (4 marks)

1. Determine whether each of the following statements is countable or incountable. Justify your answers.
   (a) $\mathbb{Q} \cap (-\infty, 2]$. **(1 mark)**
   (b) $\mathcal{P}(\mathbb{Z})$. **(1 mark)**
2. Show that the set of odd negative integers is a countable set.**(2 marks)**