[Exam's total mark =40] [Exam content: Integer Programming – Dynamic Programming – Network Analysis]

The exam consists of three parts given as follows.

Part 1: Answer only one question from the following questions (10 marks)

(**Q1**) Use Gomory's algorithm to solve the following integer problem:

$$\begin{aligned} \underset{x_1,x_2,x_3}{\text{Max}} \quad & Z = 4x_1 + 6x_2 + 2x_3 \\ \text{s.t.} \quad & 4x_1 - 4x_2 \le 5, \\ & -x_1 + 6x_2 \le 5, \\ & -x_1 + x_2 + x_3 \le 5, \\ & x_1, x_2, x_3 \ge 0, \\ & x_1 \text{ and } x_3 \text{ are integers} \end{aligned}$$

Where the optimal non-integer solution is given by the following table, $\mathbf{s_1}$, s_2 and s_3 are slack variables.

c_B	BV	x_1	x_2	x_3	s_1	s_2	s_3	b
4	x_1	1	0	0	3/10	1/5	0	5/2
6	x_2	0	1	0	1/20	1/5	0	5/4
2	x_3	0	0	1	1/4	0	1	25/4
Z_j		4	6	2	2	2	2	$Z_{max} = 30$
$\bar{c}_j$		0	0	0	-2	-2	-2	

(**Q2**) Use the Branch-and-bound (BB) technique to solve the following mixed-integer problem. (Solve only the subproblem whose constraints are of type less than or equal " $\le$ ")

$$\begin{aligned} \underset{x_1,x_2}{\text{Max}} \quad & Z = x_1 + x_2 \\ \text{s.t.} \quad & 2x_1 + 5x_2 \le 16, \\ & 6x_1 + 5x_2 \le 30, \\ & x_2 \ge 0, \\ & x_1 \ge 0 \text{ and integer} \end{aligned}$$

Where the continuous optimum solution is given in the following table,

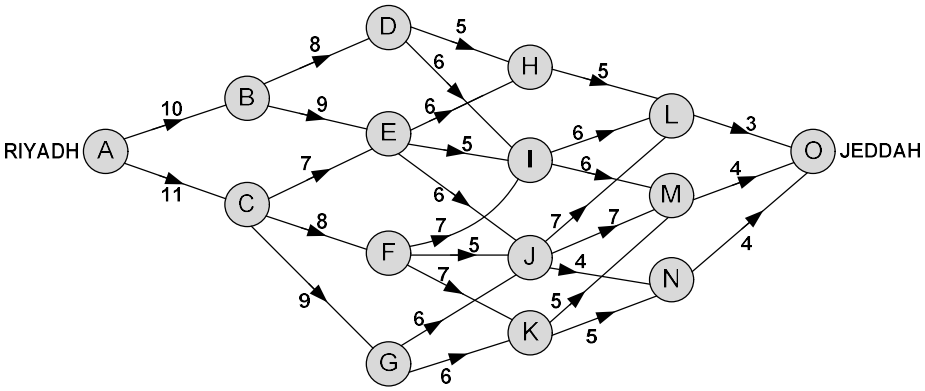
c_B	BV	x_1	x_2	s_1	s_2	b
1	x_2	0	1	3/10	-1/10	9/5
1	x_1	1	0	-1/4	1/4	7/2
Z_j		1	1	1/20	3/20	$Z_{max} = 53/10$
$\bar{c}_j$		0	0	-1/20	-3/20	

(**Q3**) Use the additive algorithm (Balas's algorithm) to solve the following 0-1 problem,

$$\begin{aligned} \underset{x_1,x_2,x_3}{\text{Max}} \quad & Z = 3x_1 + x_2 + 3x_3 \\ \text{s.t.} \quad & -x_1 + 2x_2 + x_3 \le 4, \\ & 4x_2 - 3x_3 \le 2, \\ & x_1 - 3x_2 + 2x_3 \le 3, \\ & x_1, x_2, x_3 = 0 \text{ or } 1 \end{aligned}$$

Part 2: (**Q4**) A salesman is planning a business tour from RIYADH to JEDDAH during of which he proposes to cover one city from each of the company's different market zones on route. As he/she has limited time at his disposal, he/she must complete the tour in the shortest possible time. The network given below shows the number of days' time involved for covering any of the various intermediate cities (time includes travel as well

as working time). Determine the optimum tour plan using both dynamic programming and Dijkstra's algorithm. (10 marks)



Part 3: Answer only two questions from the following questions (Each question10 marks)
Use dynamic programming for solving the following questions.

(**Q5**)

$$\text{Max} \quad \prod_{i=1}^{3} u_i$$

$$\text{s.t.} \quad \sum_{i=1}^{3} u_i = 10, \;\; u_i \geq 0; \;\; i = 1,2,3$$

Where $\prod_{i=1}^{3} u_i = u_1 u_2 u_3$

(**Q6**) In information theory, the expected amount of information is measured by Shannon-Wiener measure (or the Entropy function) given by:

$$H(p_1, p_2, \dots, p_n) = -\sum_{i=1}^{n} p_i \log_2 p_i \quad , \qquad p_i \geq 0; \; \sum_{i=1}^{n} p_i = 1$$

Solve the following problem:

$$\text{Max} \quad H(p_1, p_2, \dots, p_n)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} p_i = 1 \; ; \; p_i \geq 0$$

(**Q7**) Solve the following L.P.P.,

$$\underset{x_1, x_2}{\text{Max}} \quad Z = 4x_1 + 14x_2$$

$$\text{s.t.} \quad 2x_1 + 7x_2 \leq 21,$$
$$7x_1 + 2x_2 \leq 21,$$
$$x_1, x_2 \geq 0$$

End of Exam-Good Luck