Note:

- Write your answers in the same order as the questions.
- Write legibly.

(1) Evaluate the following integral using residues

$$\int_{-\infty}^{\infty} \frac{x\sin(x)dx}{(x^2+4)^2} \qquad \text{(6 marks}$$

(2) Evaluate the following integral using residues

$$\int_{0}^{2\pi} \frac{d\theta}{2+\sin(\theta)\cos(\theta)} \qquad \text{(6 marks)}$$

(3) Find the Laurent expansion of $f(z) = \frac{1}{z^3 - 25z}$ in the infinite annular $\{z\colon |z| > 5\}$. Use the expansion to find $\int_\gamma \frac{z^3 dz}{z^3 - 25z}$, where γ is the circle $|z| = 6$, in the positive direction. (6 marks)

(4) Let $f(z) = u(x, y) + i\, v(x, y)$ be analytic on D, show that $u_{xxyx}(x, y)$ exists in D and continuous. (5 marks)

(5) If $f(z) = u(x, y) + i\, v(x, y)$ is analytic on D, then find the Cauchy- Riemann equations on D in polar coordinates, then find $f'(z)$ in polar form. (5 marks)

(6) Find $\int_\gamma \frac{Log\, z\, dz}{z^2 - 2z + 2}$, where $Log(z)$ is the main branch of the logarithm and γ is the rectangle that joins the points $\left(\frac{1}{2}, 2i\right)$, $(2, 2i)$, $(2, -2i)$, $\left(\frac{1}{2}, -2i\right)$ in the positive direction. (6 marks)

(7) Let $f(z) = \frac{\cosh(z^2) - 1}{z^4}$. Show that $f(z)$ has a removable singularity at $z = 0$ and use the uniqueness of Taylor expansion around $z = 0$ to find $f^{(10)}(0)$. (6 marks)