KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

MATH-244 (Linear Algebra); Final Exam; Semester 1 (1443 H)

Max. Marks: 40 **Max. Time: 3 hours**

Note: ***Attempt all the five questions!***

Question 1 [4+2+2 marks]:

a) Find adjoint of the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 6 & 2 \\ -2 & 3 & 6 \end{bmatrix}$ and then find A^{-1}.

b) Evaluate $det(det(A)\, B^2 A^{-1})$, where **A** and **B** are square matrices of order 3 with $det(\mathbf{A}) = \mathbf{3}$ and $det(\mathbf{B}) = \mathbf{2}$.

c) Let $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 6 & 3 \\ 0 & 2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 4 \\ -1 & 0 & 8 \end{bmatrix}$. Explain why the matrices A and B are not row equivalent to each other?

Question 2 [5+3 marks]:

a) Find the values of α and β such that the following linear system:

$$\begin{aligned} x - 2y + 3z &= 4 \\ 2x - 3y + \alpha z &= 5 \\ 3x - 4y + 5z &= \beta \end{aligned}$$

has:

i) No solution;

ii) Infinitely many solutions.

b) Let $s_1 = 3 - 2x$, $s_2 = 2 + x$, $s_3 = 1 + x - x^2$, $s_4 = x + x^2 - x^3$. Find the values of a, b, c and d such that $1 - 6x - 3x^2 - 4x^3 = as_1 + bs_2 + cs_3 + ds_4$.

Question 3 [4+4 marks]:

a) Let $F = span\{u_1 = (1,1,1,1), u_2 = (0,1,2,1), u_3 = (1,0,-2,3),\ u_4 = (1,1,2,-2)\}$ in the Euclidean space $\mathbb{R}^4$. Then:

i) Find $dim(F)$

ii) Show that $(1,1,0,1) \notin F$.

b) Let $B = \{v_1 = (1,1,2), v_2 = (3,2,1), v_3 = (2,1,5)\}$ and $C = \{u_1, u_2, u_3\}$ be two bases for $\mathbb{R}^3$ such that

$$_BP_C = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

is the transition matrix from C to B. Find the vectors u_1, u_2 and u_3.

Question 4 [4+2+2 marks]:

a) Let w_1= (0,0,1), w_2= (0,1,1), w_3= (1,1,1) be vectors in the Euclidean space $\mathbb{R}^3$. Then:

i) Find the angle between w_1 and w_3.

ii) By applying the Gram-Schmidt process on $\{w_1, w_2, w_3\}$ to find an orthonormal basis of the Euclidean space $\mathbb{R}^3$.

b) Let T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation given by $T(x, y) = (x + 4y, 2x + 3y)$. Find:

i) *Ker*(*T*) ii) *dim* Im(*T*)

c) Let the linear transformation T : $\mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by:

$$\mathrm{T}(x, y) = (x + 2y, x - y, 3\,x + y).$$

Find matrix of the transformation $[\mathrm{T}]_{\mathrm{B}}^{\mathrm{C}}$, where B and C are the standard bases of $\mathbb{R}^2$ and $\mathbb{R}^3$, respectively.

Question 5 [4 + 4 marks]:

a) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Find eigenvalue/s of the matrix A and determine one basis of the corresponding eigenspace/s. Then, give reason for the non-diagonalizability of A.

b) Show that the matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}$ diagonalizes the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}$ and then use this fact to compute A^{-1}.

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