

Calculators are not allowed 2 pages

Question 1 : [7pts]

- Let A, B, C and D be matrices of order 3 such that $AB + AC - D = 0$,
 $|D| = 6$, $B = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ -1 & 1 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -2 & -1 \\ 1 & -1 & 3 \end{pmatrix}$.
Find $|A|$.
- Let R and S be matrices of order 3 such that $RS + R - 2I = 0$.
Find R^{-1} if $S = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 3 & 4 \\ 0 & 2 & 5 \end{pmatrix}$.
- Find a basis of the vector subspace $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} ; a - b - 2c - 3d = 0 \right\}$.

Question 2 : [5pts]

Find the values of m for which the following linear system

$$\begin{cases} x + my + 2z & = & 3 \\ 4x + (6 + m)y - mz & = & 13 - m \\ x + 2(m - 1)y + (m + 4)z & = & m + 2 \end{cases}$$

- has a unique solution.
- has infinite solutions.
- has no solution.

Question 3 : [8pts]

- Let V be a vector space and $B = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ a basis of V .
Explain why $u_1 - 2u_2 + 3u_6 \neq 5u_3 + 7u_4 - 6u_5$.
- Define $T: \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ by:

$$T(x, y, z, t) = (x - 2y + z + 3t, 2x - 3y + 2t, -x + 3z + 5t).$$

- Find the matrix of the linear transformation T with respect to the standard bases of $\mathbb{R}^4$ and $\mathbb{R}^3$.

- (b) Find a basis for kernel T .
- (c) Find a basis for Image T .

Question 4 : [7pts]

Let B and C be bases of a vector space V of dimension 3 such that ${}_C P_B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. (${}_C P_B$ is the transition matrix from the basis B to the basis C). Let $T: V \rightarrow V$ be a linear transformation with $[T]_B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 0 \end{pmatrix}$.

1. If $[v]_C = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, find $[v]_B$.
2. If $[w]_B = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$, find $[T(w)]_B$.
3. If $B = \{u_1, u_2, u_3\}$. Find the values of a, b, c such that $T(u_1) = au_1 - \frac{b}{5}u_2 + cu_3$.

Question 5 : [5pts]

1. Let F be the subspace of the Euclidean inner product space $\mathbb{R}^3$ spanned by $\{v_1 = (1, 1, 0), v_2 = (1, 1, 1)\}$. Use Gram-Schmidt process to get an orthonormal basis of F .
2. Let $\mathbb{R}^3$ be the Euclidean inner product space and $u = (1, -1, 1), v = (2, 0, -2)$ in $\mathbb{R}^3$.
 - (a) Find $\|u + v\|^2$.
 - (b) Find $\cos \theta$, if θ is the angle between the vectors u and v .

Question 6 : [8pts]

1. Compute B^{10} if $B = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.
2. Let $A = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 0 \\ 0 & 3 & -2 \end{pmatrix}$.
 - (a) Find the characteristic polynomial of A .
 - (b) Find the eigenvalues and its corresponding eigenvectors of A .
 - (c) Explain why A is not diagonalizable?