King Saud University
Department of Mathematics
Second Semester 1435-1436 H

MATH 244 (Linear Algebra)
Final Exam
Duration: 3 Hours

Student's Name	Student's ID	Group No.	Lecturer's Name

Question No.	I	II	III	IV	V	Total
Mark						

[**I**] Determine whether the following is **True** or **False**. [12 *Points*]

(1) If A is an invertible matrix, then AA^T is invertible. ( )

(2) If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & -2 \\ 2 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -2 \\ 0 & 2 & 1 \end{bmatrix}$, then $\det(B) = \det(A)$. ( )

(3) The following system of equations is linear

$$\begin{aligned} x + 3y^2 &= 1 \\ \sin x + y &= 0 \end{aligned}$$

( )

(4) If the characteristic polynomial of a 2×2 matrix A is $P(\lambda) = \lambda^2 - 1$, then A is invertible. ( )

(5) Any set containing three vectors from $\mathbb{R}^3$ is a basis for $\mathbb{R}^3$. ( )

(6) If $A = \begin{bmatrix} 2 & 0 \\ 3 & -1 \end{bmatrix}$, then the eigenvalues of A^4 are 16 and 1.

( )

OVER

(7) If V is a vector space, then $(-1)\mathbf{u} = -\mathbf{u}$ for all $\mathbf{u} \in V$. ( )

---

(8) $W = \{(x, y) \in \mathbb{R}^2, x^2 = y^2\}$ is a subspace of $\mathbb{R}^2$. ( )

---

(9) The vector $\mathbf{u} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ is a unit vector. ( )

---

(10) The inverse of an invertible upper triangular matrix is upper triangular. ( )

---

(11) If $(2X - I_2)^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$, then $X^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$. ( )

---

(12) The functions $\mathbf{f_1} = \sin x$ and $\mathbf{f_2} = x \sin x$ are linearly independent. ( )

---

(13) If the coordinate vector of $\mathbf{v}$ relative to S is $(\mathbf{v})_S = (1, -1)$ where $S = \{(5, 3), (2, 1)\}$, then $\mathbf{v} = (3, 2)$. ( )

---

(14) If $S = \{(1, 4), (2, 1)\}$ and $T = \{(1, 4), (2, 1), (3, 5)\}$, then Span$(S)$ =Span(T). ( )

---

(15) If C is a 6×7 matrix with $nullity(C) = 5$, then $nullity(C^T) = 2$. ( )

---

(16) If $m_1 \neq m_2$ in the system $\left\{ \begin{array}{l} -m_1 x_1 + x_2 = b_1 \\ -m_2 x_1 + x_2 = b_2 \end{array} \right.$, then the system has a unique solution. ( )

OVER

[II] Choose the correct answer. [6 *Points*]

(1) If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation such that $T(1,0,0) = (1,1,0)$, $T(0,1,0) = (1,0,1)$ and $T(0,0,1) = (0,1,1)$. Then $T(3,-2,4)$ equals.

(a) $(1,7,2)$ **(b)** $(5,7,7)$ **(c)** $(3,4,1)$ **(d)** None of the previous

---

(2) The values of c, if any, for which the matrix $\begin{bmatrix} c & -c & c \\ 1 & c & 1 \\ 0 & 0 & c \end{bmatrix}$ is invertible are

(a) $c \neq 0, 1$ **(b)** $c \neq 0, -1$ **(c)** $c = 0, -1$ **(d)** None of the previous

---

(3) If W =Span$\{(1,1,1),(2,2,2)\}$, then $\dim(W)$ is

(a) 0 **(b)** 1 **(c)** 2 **(d)** None of the previous

---

(4) If A and B are 4×4 matrices, then $\det(3A + 3AB) =$

(a) $3^4(\det A + \det A \cdot \det B)$ **(b)** $3^4 \det A \cdot (1 + \det B)$ **(c)** $3^4 \det A \cdot \det(I + B)$ **(d)** None of the previous

---

(5) If $T_1(x,y) = (x-y, x+y)$ and $T_2(x,y) = (4x, 3x+2y)$, then the standard matrix for $T_2 \circ T_1$ is

(a) $\begin{bmatrix} 1 & -2 \\ 7 & 2 \end{bmatrix}$ **(b)** $\begin{bmatrix} 4 & -4 \\ 5 & -1 \end{bmatrix}$ **(c)** $\begin{bmatrix} 4 & 5 \\ -4 & -1 \end{bmatrix}$ **(d)** None of the previous

---

(6) If T is the rotation about the origin, through an angle $\theta = 60°$, then $[T]$ equals

(a) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **(b)** $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **(c)** $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$ **(d)** None of the previous

OVER

(7) If $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$, then $rank(A)$ equals

(a) 0 **(b)** 1 **(c)** 2 **(d)** None of the previous

---

(8) The number of parameters in the general solution of $A\mathbf{x} = \mathbf{0}$, where A is a 5×7 matrix of rank 3, is

(a) 3 **(b)** 4 **(c)** 2 **(d)** None of the previous

---

(9) If $\mathbf{v} = (-1, 2, 1)$ and $\mathbf{u} = (0, 4, -5)$, then $2\mathbf{v} \cdot \mathbf{u}$ is

(a) $(0, 16, -10)$ **(b)** -4 **(c)** 6 **(d)** None of the previous

---

(10) Using Cramer's rule to solve the following system for y

$$\begin{aligned} x - 4y + z &= 6 \\ 4x - y + 2z &= 1 \\ 2x + 2y - 3z &= 0 \end{aligned}$$

gives

(a) $y = \dfrac{\begin{vmatrix} 6 & -4 & 1 \\ 1 & -1 & 2 \\ 0 & 2 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -4 & 1 \\ 4 & -1 & 2 \\ 2 & 2 & -3 \end{vmatrix}}$ **(b)** $y = \dfrac{\begin{vmatrix} 1 & 6 & 1 \\ 4 & 1 & 2 \\ 2 & 0 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -4 & 1 \\ 4 & -1 & 2 \\ 2 & 2 & -3 \end{vmatrix}}$ **(c)** $y = \dfrac{\begin{vmatrix} 1 & -4 & 6 \\ 4 & -1 & 1 \\ 2 & 2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & -4 & 1 \\ 4 & -1 & 2 \\ 2 & 2 & -3 \end{vmatrix}}$ **(d)** None of the previous

OVER

[III] [9 *Points*]

(a) i- **Show** that $\lambda = 2$ is an eigenvalue of $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

ii- **Find** a basis for the eigenspace of A corresponding to $\lambda = 2$

(b) i- **Compute** the eigenvalues of $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

ii- **Are** the columns of B linearly independent? **Justify** your answer.

OVER

[IV] [8 *Points*]

(a) Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where $\mathbf{v}_1 = (1, -3, 1, 1)$, $\mathbf{v}_2 = (2, -1, 1, 1)$, $\mathbf{v}_3 = (4, -7, 3, 3)$.

i- **Show** that S is not a basis for $\mathbb{R}^4$;

ii- **Find** a subset of S that forms a basis B for $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$;

iii- **Express** the vector of S which is not in B as a linear combination of vectors from B.

(b) **Find** a basis for the row space of $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

[V] [9 *Points*]

(a) **Find** the standard matrix for the composed transformation in $\mathbb{R}^3$ given by a reflection about the xy-plane, followed by a reflection about the xz-plane, followed by an orthogonal projection on the yz-plane.

(b) If T is the matrix operator $T : \mathbb{R}^3 \to \mathbb{R}^3$, defined by

$$\begin{cases} w_1 = & x_1 & +4x_2 & -x_3 \\ w_2 = & 2x_1 & +7x_2 & +x_3 \\ w_3 = & x_1 & +3x_2 & \end{cases}$$

i- **Show** that T is one-to-one.

ii- **Find** the standard matrix for T^{-1}

iii- **Compute** $T^{-1}(2, 0, 2)$.

iv- **Is** $Range(T) = \mathbb{R}^3$? **Justify** your answer.

Good Luck