

# Fatigue and Dynamic Loading

## Fatigue failure:

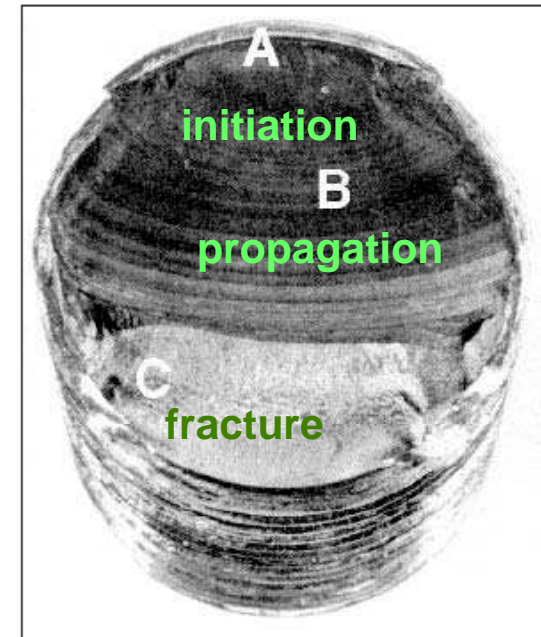
Often, machine members are found to have failed under the action of repeated or fluctuating stresses; yet the most careful analysis reveals that the actual maximum stresses were well below the ultimate strength of the material, and quite frequently even below the yield strength. The most distinguishing characteristic of these failures is that the stresses have been repeated a very large number of times. Hence the failure is called a *fatigue failure*.

When machine parts fail statically, they usually develop a very large deflection, because the stress has exceeded the yield strength, and the part is replaced before fracture actually occurs. Thus many static failures give visible warning in advance. But a fatigue failure gives no warning! It is sudden and total, and hence dangerous. It is relatively simple to design against a static failure, because our knowledge is comprehensive. Fatigue is a much more complicated phenomenon, only partially understood, and the engineer seeking competence must acquire as much knowledge of the subject as possible.

- Static conditions : loads are applied **gradually**, to give sufficient time for the strain to fully develop.
- Variable conditions : stresses **vary with time** or fluctuate between different levels, also called repeated, alternating, or fluctuating stresses.
- When machine members are found to have failed under fluctuating stresses, the **actual maximum stresses were well below the ultimate strength of the material, even below yielding strength.**
- Since these failures are due to stresses repeating for a large number of times, they are called **fatigue failures**.
- When machine parts fails statically, they usually develop a very large deflection, thus visible warning can be observed in advance; **a fatigue failure gives no warning!**

A fatigue failure has an appearance similar to a brittle fracture, as the fracture surfaces are flat and perpendicular to the stress axis with the absence of necking.

- A fatigue failure arises from three stages of development:
  - Stage I : initiation of microcracks due to cyclic plastic deformation (these cracks are not usually visible to the naked eyes).
  - Stage II : propagation of microcracks to macrocracks forming parallel plateau-like fracture surfaces separated by longitudinal ridges (in the form of dark and light bands referred to as beach marks).
  - Stage III : fracture when the remaining material cannot support the loads.



## 6-11 Characterizing Fluctuating Stresses

Fluctuating stresses in machinery often take the form of a sinusoidal pattern because of the nature of some rotating machinery. However, other patterns, some quite irregular, do occur. It has been found that in periodic patterns exhibiting a single maximum and a single minimum of force, the shape of the wave is not important, but the peaks on both the high side (maximum) and the low side (minimum) are important. Thus  $F_{\max}$  and  $F_{\min}$  in a cycle of force can be used to characterize the force pattern. It is also true that ranging above and below some baseline can be equally effective in characterizing the force pattern. If the largest force is  $F_{\max}$  and the smallest force is  $F_{\min}$ , then a steady component and an alternating component can be constructed as follows:

$$F_m = \frac{F_{\max} + F_{\min}}{2} \quad F_a = \left| \frac{F_{\max} - F_{\min}}{2} \right|$$

where  $F_m$  is the midrange steady component of force, and  $F_a$  is the amplitude of the alternating component of force.

**Figure 6-23**

Some stresstime relations:  
 (a) fluctuating stress with high-frequency ripple;  
 (b and c) nonsinusoidal fluctuating stress;  
 (d) sinusoidal fluctuating stress;  
 (e) repeated stress;  
 (f) completely reversed sinusoidal stress.

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

$$\sigma_a = \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right|$$

In addition to Eq. (6-36), the *stress ratio*

$$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

and the *amplitude ratio*

$$A = \frac{\sigma_a}{\sigma_m}$$

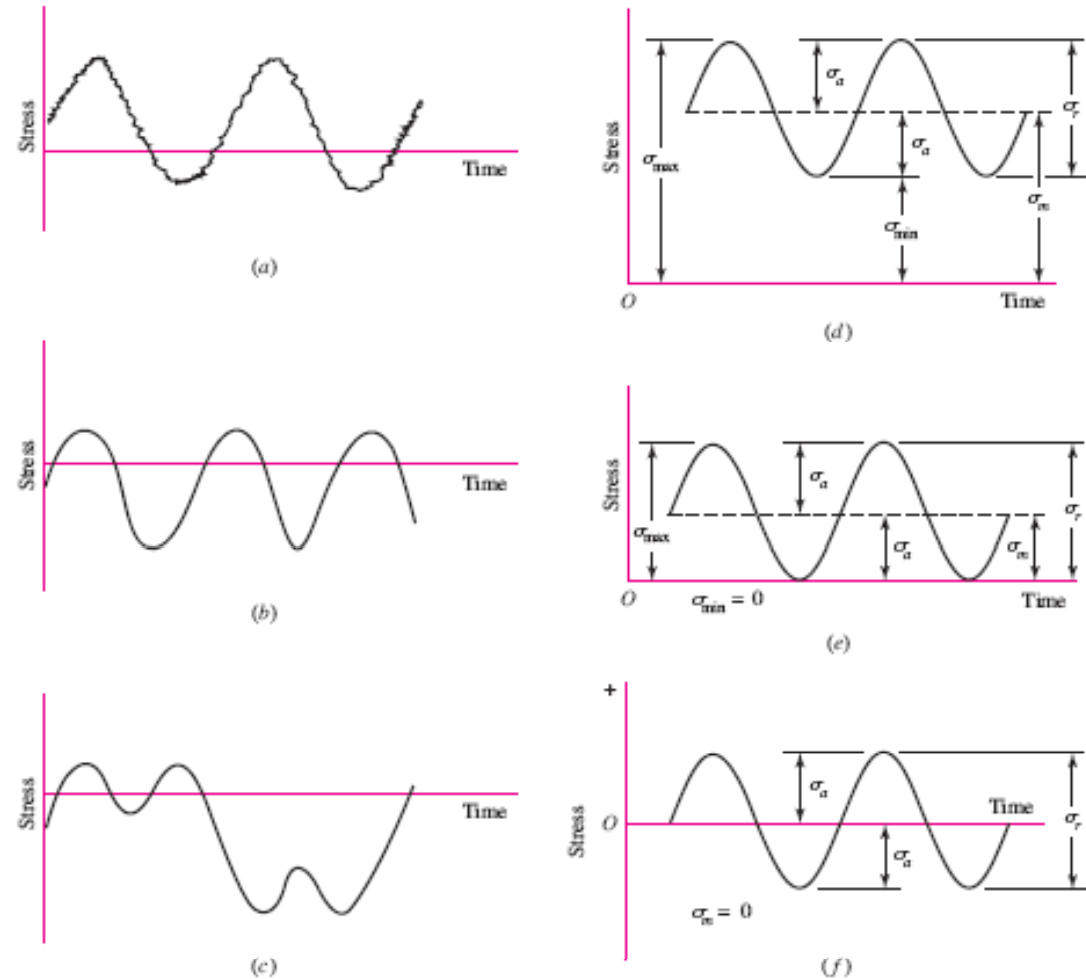


Figure 6-23 illustrates some of the various stress-time traces that occur. The components of stress, some of which are shown in Fig. 6-23d, are

$\sigma_{\min}$  = minimum stress

$\sigma_{\max}$  = maximum stress

$\sigma_a$  = amplitude component

$\sigma_m$  = midrange component

$\sigma_r$  = range of stress

$\sigma_s$  = static or steady stress

# Fatigue Life Methods in Fatigue Failure Analysis

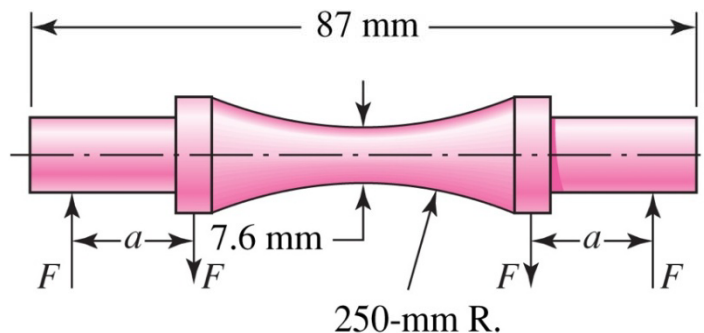
- Let  $N$  be the number of cycles to fatigue for a specified level of loading
  - For  $1 \leq N \leq 10^3$  , generally classified as low-cycle fatigue
  - For  $N > 10^3$  , generally classified as high-cycle fatigue
- Three major fatigue life methods used in design and analysis are
  - ü stress-life method : is based on stress only, least accurate especially for low-cycle fatigue; however, it is the most traditional and easiest to implement for a wide range of applications.
  - ü strain-life method : involves more detailed analysis, especially good for low-cycle fatigue; however, idealizations in the methods make it less practical when uncertainties are present.
  - ü linear-elastic fracture mechanics method : assumes a crack is already present. Practical with computer codes in predicting in crack growth with respect to stress intensity factor

## 6-2 Approach to Fatigue Failure in Analysis and Design

In this chapter, we will take a structured approach in the design against fatigue failure. As with static failure, we will attempt to relate to test results performed on simply loaded specimens. However, because of the complex nature of fatigue, there is much more to account for. From this point, we will proceed methodically, and in stages. In an attempt to provide some insight as to what follows in this chapter, a brief description of the remaining sections will be given here.

## 6-4 The Stress-Life Method

To determine the strength of materials under the action of fatigue loads, specimens are subjected to repeated or varying forces of specified magnitudes while the cycles or stress reversals are counted to destruction. The most widely used fatigue-testing device

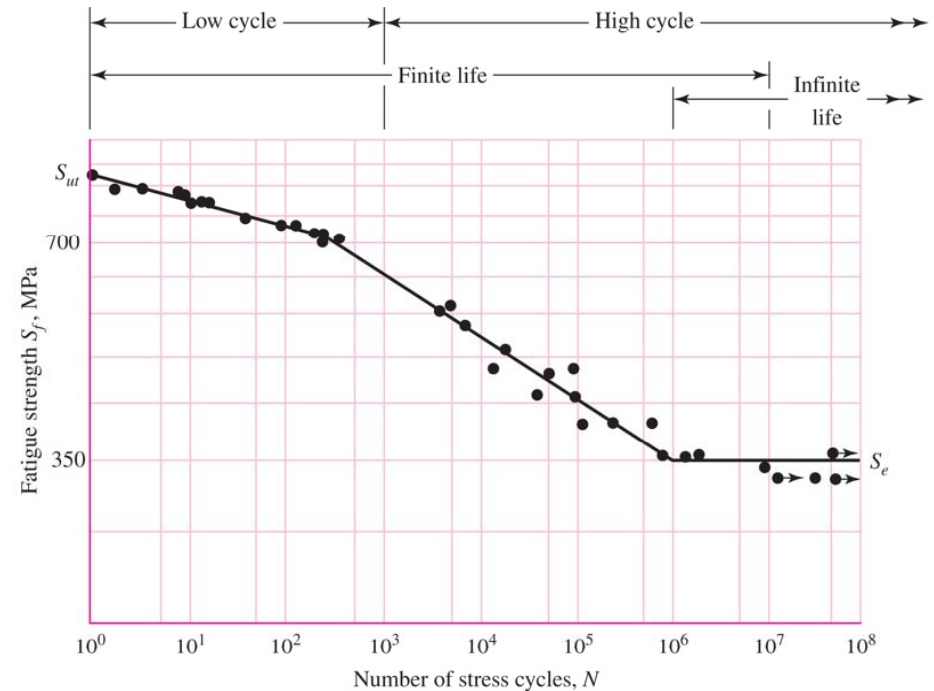


- The most widely used fatigue-testing device is the R. R. Moore high-speed rotating-beam machine.
- Specimens in R.R. Moore machines are subjected to pure bending by means of added weights.
- Other fatigue-testing machines are available for applying fluctuating or reversed axial stresses, torsional stresses, or combined stresses to the test specimens.



# S-N Curve

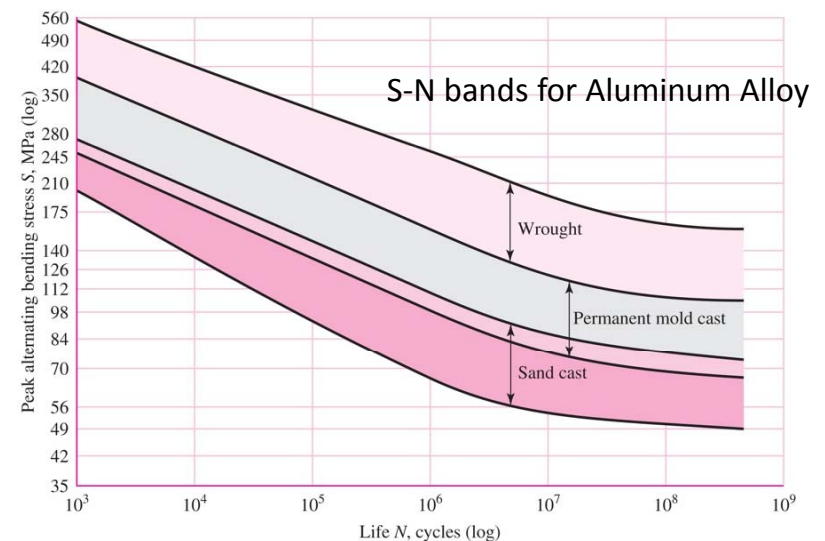
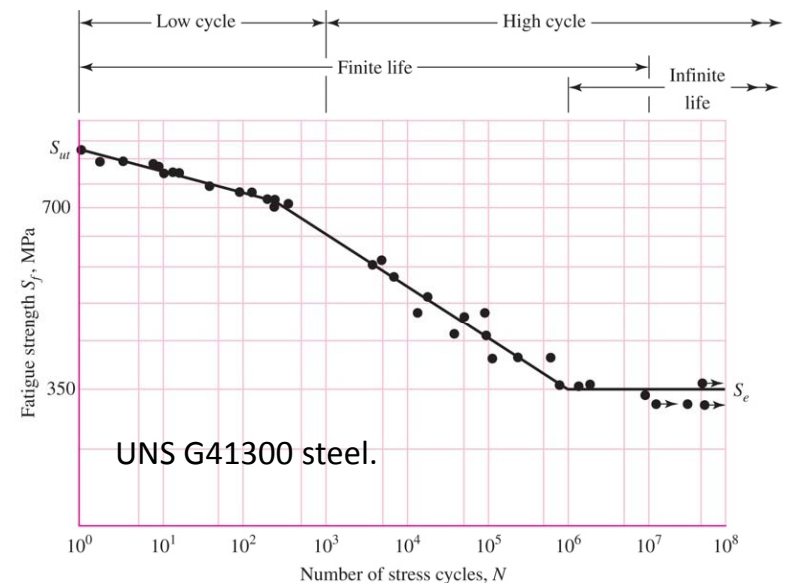
- In R. R. Moore machine tests, a constant bending load is applied, and the number of revolutions of the beam required for failure is recorded.
- Tests at various bending stress levels are conducted.
- These results are plotted as an **S-N** diagram.
- Log plot is generally used to emphasize the bend in the S-N curve.
- Ordinate of S-N curve is **fatigue strength,  $S_f$** , at a specific number of cycles of cycles



S-N diagram from the results of completely reversed axial fatigue test. Material : UNS G41300 steel.

# Characteristics of S-N Curves for Metals

- In the case of steels, a knee occurs in the graph, and beyond this knee failure will not occur, no matter how great the number of cycles - this knee is called the *endurance limit*, denoted as  $S_e$
- Non-ferrous metals and alloys do not have an endurance limit, since their S-N curve never become horizontal.
- For materials with no endurance limit, the fatigue strength is normally reported at  $N = 5 \times 10^8$
- $N = 1/2$  is the simple tension test



● We note that a stress cycle ( $N = 1$ ) constitutes a single application and removal of a load and then another application and removal of the load in the opposite direction. Thus  $N = \frac{1}{2}$  means the load is applied once and then removed, which is the case with the simple tension test.

● The body of knowledge available on fatigue failure from  $N = 1$  to  $N = 1000$  cycles is generally classified as *low-cycle fatigue*, as indicated in Fig. 6–10. *High-cycle fatigue*, then, is concerned with failure corresponding to stress cycles greater than  $10^3$  cycles.

● We also distinguish a *finite-life region* and an *infinite-life region* in Fig. 6–10. The boundary between these regions cannot be clearly defined except for a specific material; but it lies somewhere between  $10^6$  and  $10^7$  cycles for steels, as shown in Fig. 6–10.

● As stated earlier, the stress-life method is the least accurate approach especially for low-cycle applications. However, it is the most traditional method, with much published data available. It is the easiest to implement for a wide range of design applications and represents high-cycle applications adequately. For these reasons the stress-life method will be emphasized in subsequent sections of this chapter.

## 6-5 The Strain-Life Method

$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2}$$

we have for the total-strain amplitude

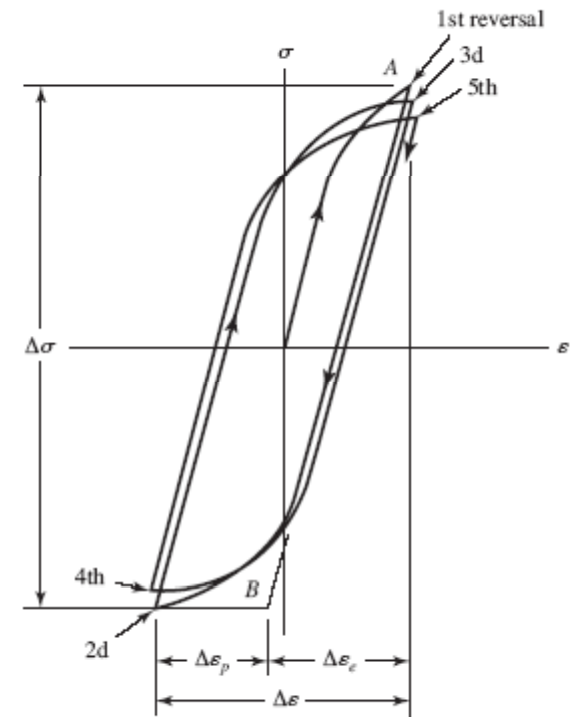
$$\frac{\Delta \varepsilon}{2} = \frac{\sigma'_F}{E} (2N)^b + \varepsilon'_F (2N)^c$$

which is the Manson-Coffin relationship between fatigue life and total strain.<sup>5</sup> Some values of the coefficients and exponents are listed in Table A-23. Many more are included in the SAE J1099 report.<sup>6</sup>

- *Fatigue ductility coefficient*  $\varepsilon'_F$  is the true strain corresponding to fracture in one reversal (point A in Fig. 6-12). The plastic-strain line begins at this point in Fig. 6-13.
- *Fatigue strength coefficient*  $\sigma'_F$  is the true stress corresponding to fracture in one reversal (point A in Fig. 6-12). Note in Fig. 6-13 that the elastic-strain line begins at  $\sigma'_F/E$ .
- *Fatigue ductility exponent*  $c$  is the slope of the plastic-strain line in Fig. 6-13 and is the power to which the life  $2N$  must be raised to be proportional to the true plastic-strain amplitude. If the number of stress reversals is  $2N$ , then  $N$  is the number of
- *Fatigue strength exponent*  $b$  is the slope of the elastic-strain line, and is the power to which the life  $2N$  must be raised to be proportional to the true-stress amplitude.

**Figure 6-12**

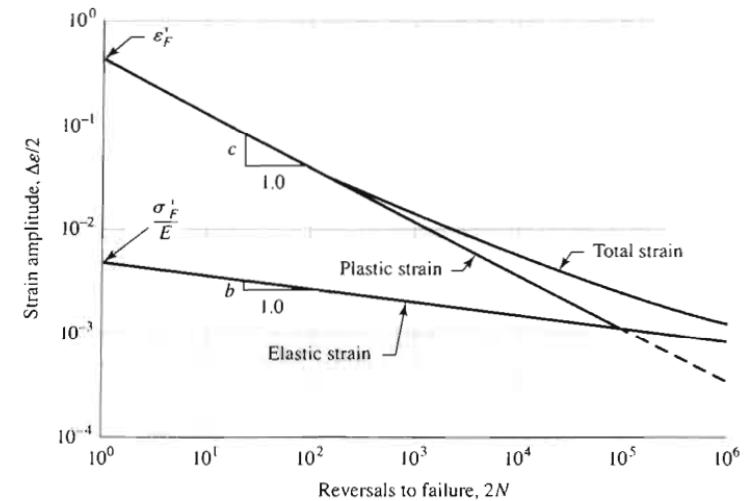
True stress-true strain hysteresis loops showing the first five stress reversals of a cyclic-softening material. The graph is slightly exaggerated for clarity. Note that the slope of the line AB is the modulus of elasticity  $E$ . The stress range is  $\Delta \sigma$ ,  $\Delta \varepsilon_p$  is the plastic-strain range, and  $\Delta \varepsilon_e$  is the elastic-strain range. The total-strain range is  $\Delta \varepsilon = \Delta \varepsilon_p + \Delta \varepsilon_e$ .



# Manson-Coffin Relationship

- The total-strain amplitude is the sum of elastic and plastic strain

$$\begin{aligned}\frac{\Delta \epsilon}{2} &= \frac{\Delta \epsilon_e}{2} + \frac{\Delta \epsilon_p}{2} \\ &= \frac{\sigma'_F}{E} (2N)^b + \epsilon'_F (2N)^c\end{aligned}$$



- $\sigma'_F$  is the fatigue strength coefficient, the true stress corresponding to fracture in one reversal.
- $b$  is the fatigue strength exponent as the slope of the elastic-strain line.
- $\epsilon'_F$  is the fatigue ductility coefficient, the true strain corresponding to fracture in one reversal.
- $c$  is the fatigue strength exponent as the slope of the plastic-strain line.

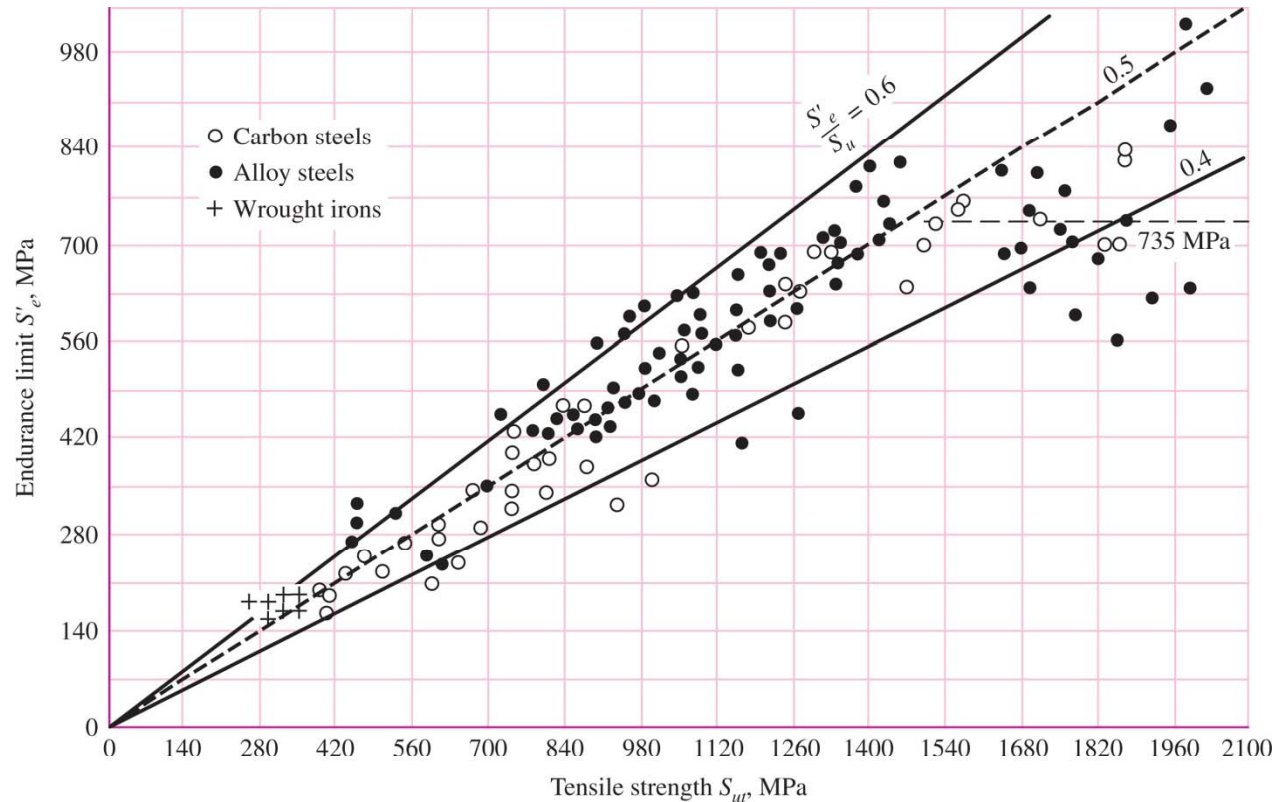
**Table A-23**

Mean Monotonic and Cyclic Stress-Strain Properties of Selected Steels Source: ASM Metals Reference Book, 2nd ed., American Society for Metals, Metals Park, Ohio, 1983, p. 217.

Grade (a)	Orientation (e)	Description (f)	Hardness HB	Tensile Strength $S_{ut}$		Reduction in Area %	True Strain at Fracture $\epsilon_f$	Modulus of Elasticity $E$		Fatigue Strength Coefficient $a'$		Fatigue Strength Exponent $b$	Fatigue Ductility Coefficient $a'_f$	Fatigue Ductility Exponent $c$
				MPa	ksi			GPa	$10^6$ psi	MPa	ksi			
A538A [b]	L	STA	405	1515	220	67	1.10	185	27	1655	240	-0.065	0.30	-0.62
A538B [b]	L	STA	460	1860	270	56	0.82	185	27	2135	310	-0.071	0.80	-0.71
A538C [b]	L	STA	480	2000	290	55	0.81	180	26	2240	325	-0.07	0.60	-0.75
AM350 [c]	L	HR, A		1315	191	52	0.74	195	28	2800	406	-0.14	0.33	-0.84
AM350 [c]	L	CD	496	1905	276	20	0.23	180	26	2690	390	-0.102	0.10	-0.42
Galnex [c]	LT	HR sheet		530	77	58	0.86	200	29.2	805	117	-0.07	0.86	-0.65
Galnex [c]	L	HR sheet		510	74	64	1.02	200	29.2	805	117	-0.071	0.86	-0.68
H11	L	Auxformed	660	2585	375	33	0.40	205	30	3170	460	-0.077	0.08	-0.74
RQC100 [c]	LT	HR plate	290	940	136	43	0.56	205	30	1240	180	-0.07	0.66	-0.69
RQC100 [c]	L	HR plate	290	930	135	67	1.02	205	30	1240	180	-0.07	0.66	-0.69
10B62	L	Q&T	430	1640	238	38	0.89	195	28	1780	258	-0.067	0.32	-0.56
1005-1009	LT	HR sheet	90	360	52	73	1.3	205	30	580	84	-0.09	0.15	-0.43
1005-1009	LT	CD sheet	125	470	68	66	1.09	205	30	515	75	-0.059	0.30	-0.51
1005-1009	L	CD sheet	125	415	60	64	1.02	200	29	540	78	-0.073	0.11	-0.41
1005-1009	L	HR sheet	90	345	50	80	1.6	200	29	640	93	-0.109	0.10	-0.39
1015	L	Normalized	80	415	60	68	1.14	205	30	825	120	-0.11	0.95	-0.64
1020	L	HR plate	108	440	64	62	0.96	205	29.5	895	130	-0.12	0.41	-0.51
1040	L	As forged	225	620	90	60	0.93	200	29	1540	223	-0.14	0.61	-0.57
1045	L	Q&T	225	725	105	65	1.04	200	29	1225	178	-0.095	1.00	-0.66
1045	L	Q&T	410	1450	210	51	0.72	200	29	1860	270	-0.073	0.60	-0.70
1045	L	Q&T	390	1345	195	59	0.89	205	30	1585	230	-0.074	0.45	-0.68
1045	L	Q&T	450	1585	230	55	0.81	205	30	1795	260	-0.07	0.35	-0.69
1045	L	Q&T	500	1825	265	51	0.71	205	30	2275	330	-0.08	0.25	-0.68
1045	L	Q&T	595	2240	325	41	0.52	205	30	2725	395	-0.081	0.07	-0.60
1144	L	CD5R	265	930	135	33	0.51	195	28.5	1000	145	-0.08	0.32	-0.58

## 6-7 The Endurance Limit

The determination of endurance limits by fatigue testing is now routine, though a lengthy procedure. Generally, stress testing is preferred to strain testing for endurance limits.



**Figure 6-17**

Graph of endurance limits versus tensile strengths from actual test results for a large number of wrought irons and steels. Ratios of  $S'_e/S_{ut}$  of 0.60, 0.50, and 0.40 are shown by the solid and dashed lines. Note also the horizontal dashed line for  $S'_e = 735$  MPa. Points shown having a tensile strength greater than 1470 MPa have a mean endurance limit of  $S'_e = 735$  MPa and a standard deviation of 95 MPa. (*Collated from data compiled by H. J. Grover, S. A. Gordon, and L. R. Jackson in Fatigue of Metals and Structures, Bureau of Naval Weapons Document NAVWEPS 00-25-534, 1960; and from Fatigue Design Handbook, SAE, 1968, p. 42.*)

$$S'_e = \begin{cases} 0.5S_{ut}, & S_{ut} \leq 1400\text{MPa} \\ 700\text{MPa}, & S_{ut} > 1400\text{MPa} \end{cases}$$

Steels treated to give different microstructures have different  $S'_e/S_{ut}$  ratios. It appears that the more ductile microstructures have a higher ratio. Martensite has a very brittle nature and is highly susceptible to fatigue-induced cracking; thus the ratio is low. When designs include detailed heat-treating specifications to obtain specific microstructures, it is possible to use an estimate of the endurance limit based on test data for the particular microstructure; such estimates are much more reliable and indeed should be used.

The endurance limits for various classes of cast irons, polished or machined, are given in Table A-24. Aluminum alloys do not have an endurance limit. The fatigue strengths of some aluminum alloys at  $5(10^8)$  cycles of reversed stress are given in Table A-24.



## Table A-24

### Mechanical Properties of Three Non-Steel Metals

#### (a) Typical Properties of Gray Cast Iron

[The American Society for Testing and Materials (ASTM) numbering system for gray cast iron is such that the numbers correspond to the *minimum tensile strength* in kpsi. Thus an ASTM No. 20 cast iron has a minimum tensile strength of 138 MPa (20 kpsi). Note particularly that the tabulations are *typical* of several heats.]

ASTM Number	Tensile Strength $S_{ut}$ , MPa (kpsi)	Compressive Strength $S_{uc}$ , MPa (kpsi)	Shear Modulus of Rupture $S_{su}$ , MPa (kpsi)	Modulus of Elasticity, Mpsi		Endurance Limit* $S_e$ , MPa (kpsi)	Brinell Hardness $H_B$	Fatigue Stress-Concentration Factor $K_f$
				Tension <sup>†</sup>	Torsion			
20	152 (22)	572 (83)	179 (26)	9.6–14	3.9–5.6	69 (10)	156	1.00
25	179 (26)	669 (97)	220 (32)	11.5–14.8	4.6–6.0	79 (11.5)	174	1.05
30	214 (31)	752 (109)	276 (40)	13–16.4	5.2–6.6	97 (14)	201	1.10
35	252 (36.5)	855 (124)	334 (48.5)	14.5–17.2	5.8–6.9	110 (16)	212	1.15
40	293 (42.5)	970 (140)	393 (57)	16–20	6.4–7.8	128 (18.5)	235	1.25
50	362 (52.5)	1130 (164)	503 (73)	18.8–22.8	7.2–8.0	148 (21.5)	262	1.35
60	431 (62.5)	1293 (187.5)	610 (88.5)	20.4–23.5	7.8–8.5	169 (24.5)	302	1.50

\*Polished or machined specimens.

<sup>†</sup>The modulus of elasticity of cast iron in compression corresponds closely to the upper value in the range given for tension and is a more constant value than that for tension.

**Table A-24**

Mechanical Properties of Three Non-Steel Metals (*Continued*)

(b) Mechanical Properties of Some Aluminum Alloys

[These *are typical* properties for sizes of about  $\frac{1}{2}$  in; similar properties can be obtained by using proper purchase specifications. The values given for fatigue strength correspond to  $50(10^7)$  cycles of completely reversed stress. Aluminum alloys do not have an endurance limit. Yield strengths were obtained by the 0.2 percent offset method.]

Aluminum Association Number	Temper	Yield, $S_y$ , MPa (kpsi)	Strength Tensile, $S_{ut}$ , MPa (kpsi)	Fatigue, $S_f$ , MPa (kpsi)	Elongation in 2 in, %	Brinell Hardness $H_B$
Wrought:						
2017	O	70 (10)	179 (26)	90 (13)	22	45
2024	O	76 (11)	186 (27)	90 (13)	22	47
	T3	345 (50)	482 (70)	138 (20)	16	120
3003	H12	117 (17)	131 (19)	55 (8)	20	35
	H16	165 (24)	179 (26)	65 (9.5)	14	47
3004	H34	186 (27)	234 (34)	103 (15)	12	63
	H38	234 (34)	276 (40)	110 (16)	6	77
5052	H32	186 (27)	234 (34)	117 (17)	18	62
	H36	234 (34)	269 (39)	124 (18)	10	74
Cast:						
319.0*	T6	165 (24)	248 (36)	69 (10)	2.0	80
333.0†	T5	172 (25)	234 (34)	83 (12)	1.0	100
	T6	207 (30)	289 (42)	103 (15)	1.5	105
335.0*	T6	172 (25)	241 (35)	62 (9)	3.0	80
	T7	248 (36)	262 (38)	62 (9)	0.5	85

\*Sand casting.

†Permanent-mold casting.

(c) Mechanical Properties of Some Titanium Alloys

Titanium Alloy	Condition	Yield, $S_y$ (0.2% offset) MPa (kpsi)	Strength Tensile, $S_{ut}$ MPa (kpsi)	Elongation in 2 in, %	Hardness (Brinell or Rockwell)
Ti-35A <sup>†</sup>	Annealed	210 (30)	275 (40)	30	135 HB
Ti-50A <sup>†</sup>	Annealed	310 (45)	380 (55)	25	215 HB
Ti-0.2 Pd	Annealed	280 (40)	340 (50)	28	200 HB
Ti-5 Al-2.5 Sn	Annealed	760 (110)	790 (115)	16	36 HRC
Ti-8 Al-1 Mo-1 V	Annealed	900 (130)	965 (140)	15	39 HRC
Ti-6 Al-6 V-2 Sn	Annealed	970 (140)	1030 (150)	14	38 HRC
Ti-6Al-4V	Annealed	830 (120)	900 (130)	14	36 HRC
Ti-13 V-11 Cr-3 Al	Sol. + aging	1207 (175)	1276 (185)	8	40 HRC

<sup>†</sup>Commercially pure alpha titanium.

# Fatigue Strength : Basics

- Low-cycle fatigue considers the range from  $N=1$  to about 1000 cycles.
- In this region, the fatigue strength  $s_f$  is only slightly smaller than the tensile strength  $s_{ut}$ .
- High-cycle fatigue domain extends from  $10^3$  to the endurance limit life ( $10^6$  to  $10^7$  cycles).
- Experience has shown that high-cycle fatigue data are rectified by a logarithmic transform to both stress and cycles-to-failure.

# Fatigue Strength at Different $N$

- Define the fatigue strength at a specified number of cycles as  $(S'_f)_N$

- By combining the elastic strain relations, we can get

$$(S'_f)_N = E \Delta \varepsilon_e / 2, \quad (S'_f)_N = \sigma'_F (2N)^b$$

- Define  $f$  as the fraction of tensile strength to  $(S'_f)_{10^3}$ . The value of  $f$  at  $10^3$  cycles is then

$$f = \frac{\sigma'_F}{S_{ut}} (2 \cdot 10^3)^b$$

- To find  $b$ , substitute the endurance strength  $(S'_e)$  and the corresponding cycles  $(N_e)$  and solving for  $b$  as

$$b = - \frac{\log(\sigma'_F / S'_e)}{\log(2N_e)}$$

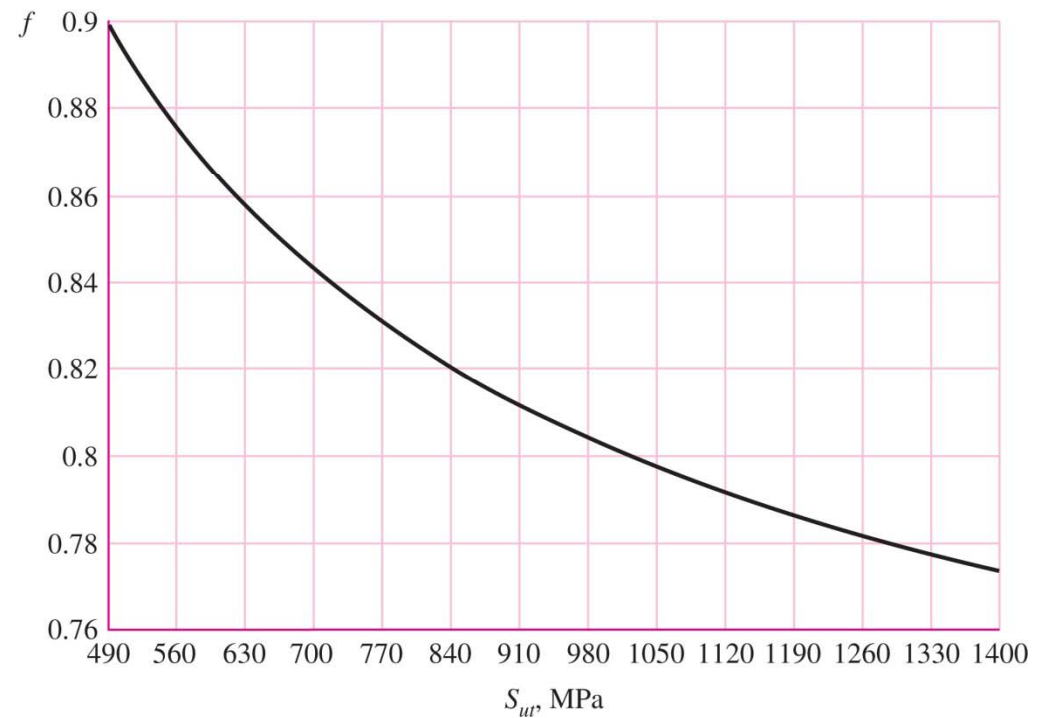
- For example, for steels when

$$\begin{array}{ll} \sigma'_F = S_{ut} + 345 & S'_e = 366 \text{MPa} \\ S_{ut} = 735 \text{MPa} & N_e = 10^6 \end{array} \quad \longrightarrow \quad S'_f = 1030 N^{-0.0749}$$

# Fatigue Strength and estimates of fatigue life

**Figure 6-18**

Fatigue strength fraction,  $f$ ,  
of  $S_{ut}$  at  $10^3$  cycles for  
 $S_e = S'_e = 0.5S_{ut}$  at  $10^6$  cycles.



$490 \text{ MPa} < S_{ut} < 1400 \text{ MPa}$  ( use the plot) ,  $S_{ut} < 350 \text{ MPa}$ ,  $f=0.9$

$$S_f = a N^b \quad (6-13)$$

where  $N$  is cycles to failure and the constants  $a$  and  $b$  are defined by the points  $10^3, (S_f)_{10^3}$  and  $10^6, S_e$  with  $(S_f)_{10^3} = f S_{ut}$ . Substituting these two points in Eq. (6-13) gives

$$a = \frac{(f S_{ut})^2}{S_e} \quad (6-14)$$

$$b = -\frac{1}{3} \log \left( \frac{f S_{ut}}{S_e} \right) \quad (6-15)$$

$$N = \left( \frac{\sigma_a}{a} \right)^{1/b}$$

**EXAMPLE 6-2**

Given a 1050 HR steel, *estimate*

- (a) the rotating-beam endurance limit at  $10^6$  cycles.
- (b) the endurance strength of a polished rotating-beam specimen corresponding to  $10^4$  cycles to failure
- (c) the expected life of a polished rotating-beam specimen under a completely reversed stress of 385 MPa.

**Solution**

(a) From Table A-20,  $S_{ut} = 630$  MPa. From Eq. (6-8),

**Answer**

$$S'_e = 0.5(630) = 315 \text{ MPa}$$

(b) From Fig. 6-18, for  $S_{ut} = 630$  MPa,  $f \doteq 0.86$ . From Eq. (6-14),

$$a = \frac{[0.86(630)^2]}{315} = 1084 \text{ MPa}$$

From Eq. (6-15),

$$b = -\frac{1}{3} \log \left[ \frac{0.86(630)}{315} \right] = -0.0785$$

Thus, Eq. (6-13) is

$$S'_f = 1084 N^{-0.0785}$$

**Answer**

For  $10^4$  cycles to failure,  $S'_f = 1084(10^4)^{-0.0785} = 526 \text{ MPa}$

(c) From Eq. (6-16), with  $\sigma_a = 385$  MPa,

**Answer**

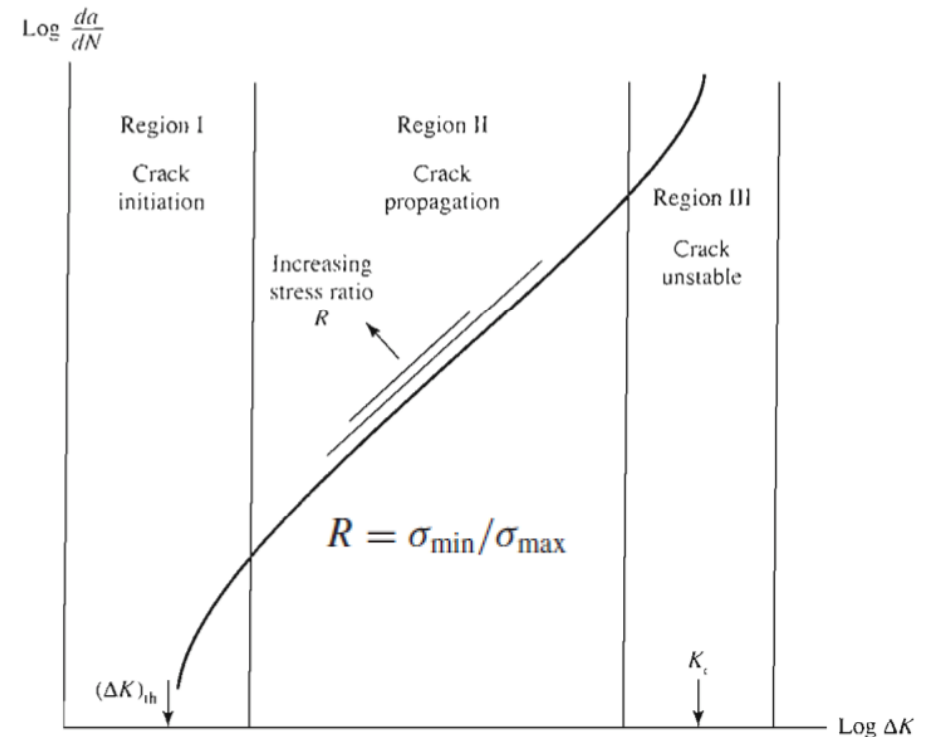
$$N = \left( \frac{385}{1084} \right)^{1/-0.0785} = 53.3(10^4) \text{ cycles}$$

Keep in mind that these are only *estimates*. So expressing the answers using three-place accuracy is a little misleading.

# Linear-Elastic Fracture Mechanics Method

- Fatigue cracking consists three stages

- ✓ Stage I : crack initiation, invisible to the observer.
- ✓ Stage II : crack propagation, most of a crack's life
- ✓ Stage III : final fracture due to rapid acceleration of crack growth.



stress intensity is given by  $K_I = \beta \sigma \sqrt{\pi a}$ . Thus, for  $\Delta \sigma$ , the stress intensity range per cycle is

$$\Delta K_I = \beta(\sigma_{max} - \sigma_{min})\sqrt{\pi a} = \beta \Delta \sigma \sqrt{\pi a} \quad (6-4)$$



# Paris Law for Crack Growth

- Assuming a crack is discovered early in stage II, the crack growth can be approximated by the Paris equation as
- $\Delta K_I$  is the variation in stress intensity factor due to fluctuating stresses.

$$\frac{da}{dN} = C(\Delta K_I)^m$$

crack length

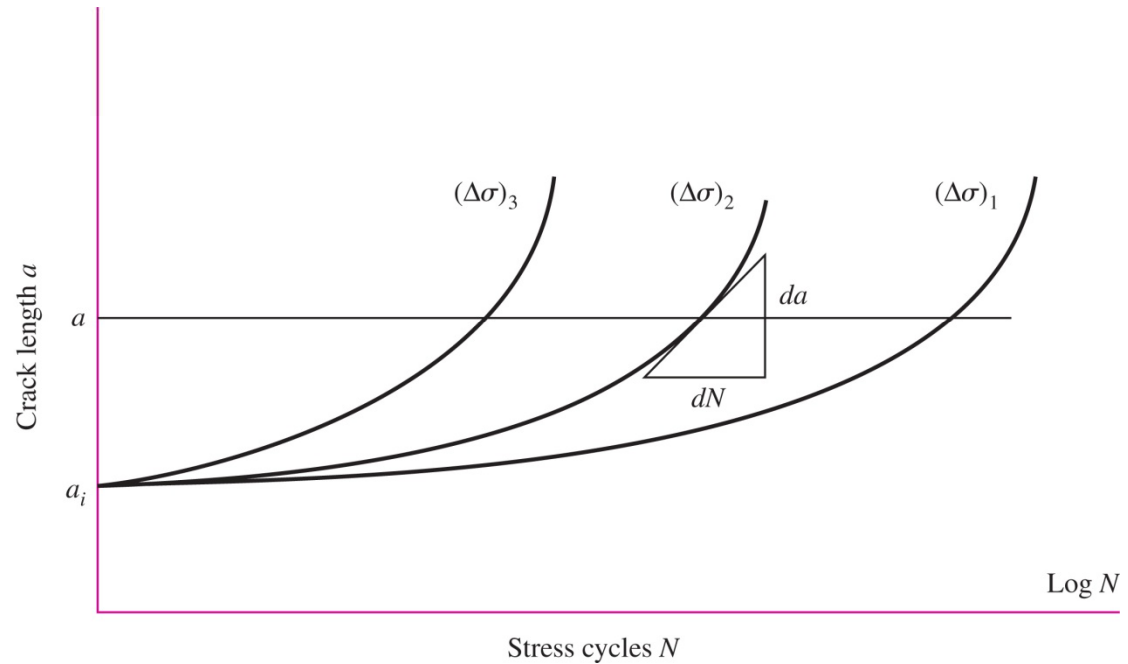
number of cycles

material constants

$$\Delta K_I = \beta(\sigma_{\max} - \sigma_{\min})\sqrt{\pi a}$$

**Figure 6-14**

The increase in crack length  $a$  from an initial length of  $a_i$  as a function of cycle count for three stress ranges,  $(\Delta\sigma)_3 > (\Delta\sigma)_2 > (\Delta\sigma)_1$ .



**Table 6-1**

Conservative Values of Factor  $C$  and Exponent  $m$  in Eq. (6-5) for Various Forms of Steel ( $R = \sigma_{\max}/\sigma_{\min} \doteq 0$ )

Material	$C, \frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$	$C, \frac{\text{in/cycle}}{(\text{kpsi}\sqrt{\text{in}})^m}$	$m$
Ferritic-pearlitic steels	$6.89(10^{-12})$	$3.60(10^{-10})$	3.00
Martensitic steels	$1.36(10^{-10})$	$6.60(10^{-9})$	2.25
Austenitic stainless steels	$5.61(10^{-12})$	$3.00(10^{-10})$	3.25

From J. M. Barsom and S. T. Rolfe, *Fatigue and Fracture Control in Structures*, 2nd ed., Prentice Hall, Upper Saddle River, NJ, 1987, pp. 288–291, Copyright ASTM International. Reprinted with permission.

$$\frac{da}{dN} = C(\Delta K_I)^m \quad (6-5)$$

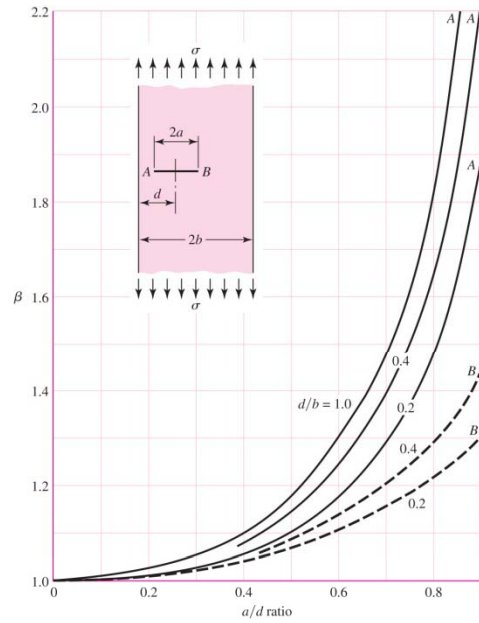
where  $C$  and  $m$  are empirical material constants and  $\Delta K_I$  is given by Eq. (6-4). Representative, but conservative, values of  $C$  and  $m$  for various classes of steels are listed in Table 6-1. Substituting Eq. (6-4) and integrating gives

$$\int_0^{N_f} dN = N_f = \frac{1}{C} \int_{a_i}^{a_f} \frac{da}{(\beta \Delta \sigma \sqrt{\pi a})^m} \quad (6-6)$$

Here  $a_i$  is the initial crack length,  $a_f$  is the final crack length corresponding to failure, and  $N_f$  is the estimated number of cycles to produce a failure *after* the initial crack is formed. Note that  $\beta$  may vary in the integration variable (e.g., see Figs. 5-25 to 5-30).

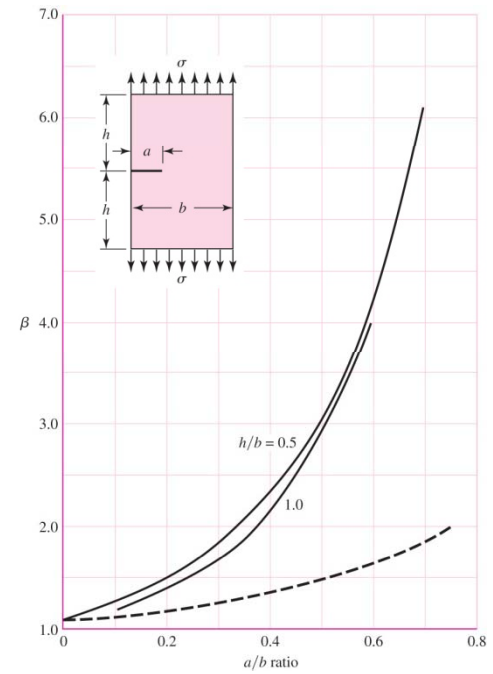
**Figure 5-25**

Off-center crack in a plate in longitudinal tension; solid curves are for the crack tip at A; dashed curves are for the tip at B.



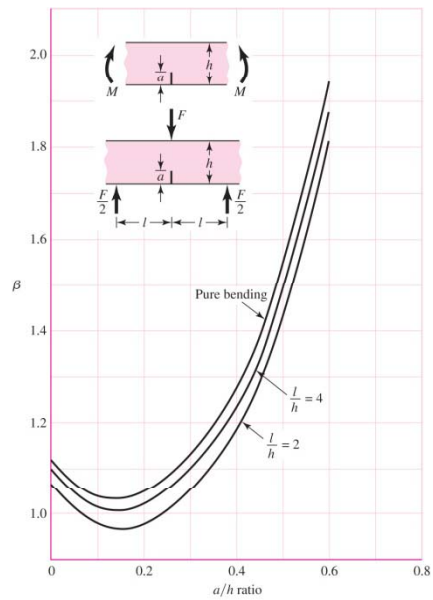
**Figure 5-26**

Plate loaded in longitudinal tension with a crack at the edge; for the solid curve there are no constraints to bending; the dashed curve was obtained with bending constraints added.



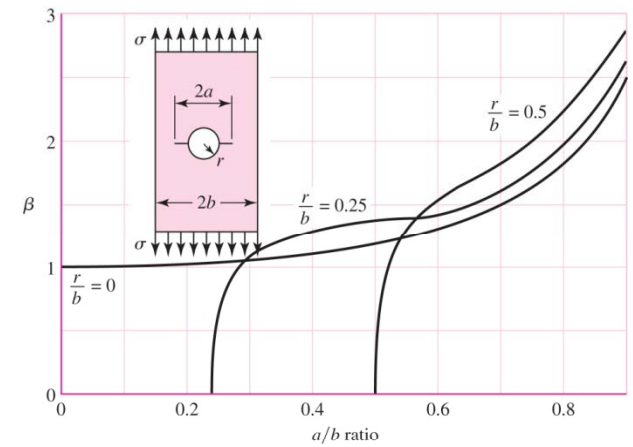
**Figure 5-27**

Beams of rectangular cross section having an edge crack.



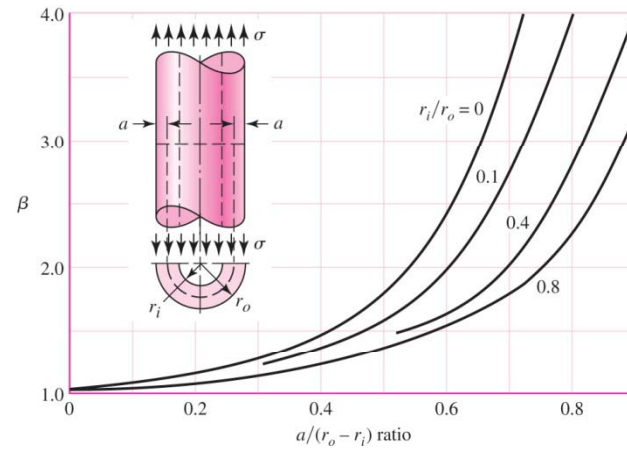
**Figure 5-28**

Plate in tension containing a circular hole with two cracks.



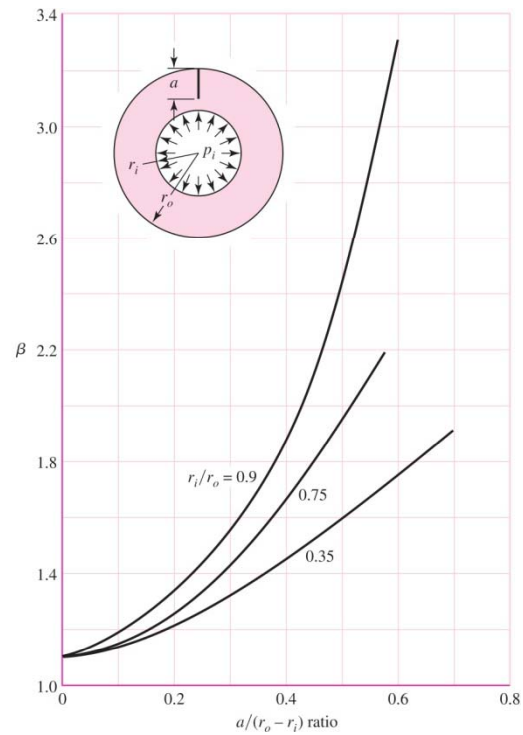
**Figure 5-29**

A cylinder loading in axial tension having a radial crack of depth  $a$  extending completely around the circumference of the cylinder.



**Figure 5-30**

Cylinder subjected to internal pressure  $p$ , having a radial crack in the longitudinal direction of depth  $a$ . Use Eq. (4-51) for the tangential stress at  $r = r_o$ .



### EXAMPLE 6-1

The bar shown in Fig. 6-16 is subjected to a repeated moment  $0 \leq M \leq 135 \text{ N} \cdot \text{m}$ . The bar is AISI 4430 steel with  $S_{ut} = 1.28 \text{ GPa}$ ,  $S_y = 1.17 \text{ GPa}$ , and  $K_{Ic} = 81 \text{ MPa} \sqrt{\text{m}}$ . Material tests on various specimens of this material with identical heat treatment indicate worst-case constants of  $C = 114 \times 10^{-15} (\text{m/cycle})/(\text{MPa} \sqrt{\text{m}})^m$  and  $m = 3.0$ . As shown, a nick of size  $0.1 \text{ mm}$  has been discovered on the bottom of the bar. Estimate the number of cycles of life remaining.

### Solution

The stress range  $\Delta\sigma$  is always computed by using the nominal (uncracked) area. Thus

$$\frac{I}{c} = \frac{bh^2}{6} = \frac{0.006(0.012)^2}{6} = 144 \times 10^{-9} \text{ m}^3$$

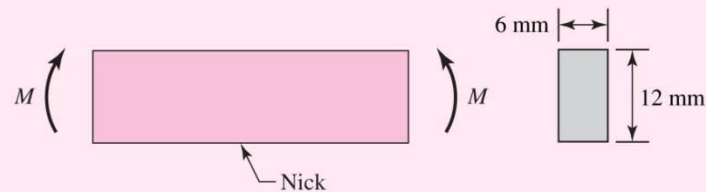
Therefore, before the crack initiates, the stress range is

$$\Delta\sigma = \frac{\Delta M}{I/c} = \frac{135}{144 \times 10^{-9}} = 937.5 \text{ MPa}$$

which is below the yield strength. As the crack grows, it will eventually become long enough such that the bar will completely yield or undergo a brittle fracture. For the ratio of  $S_y/S_{ut}$  it is highly unlikely that the bar will reach complete yield. For brittle fracture, designate the crack length as  $a_f$ . If  $\beta = 1$ , then from Eq. (5-37) with  $K_I = K_{Ic}$ , we approximate  $a_f$  as

$$a_f = \frac{1}{\pi} \left( \frac{K_{Ic}}{\beta\sigma_{\max}} \right)^2 = \frac{1}{\pi} \left( \frac{81}{937.5} \right)^2 = 0.0024 \text{ m}$$

| Figure 6-16



From Fig. 5–27, we compute the ratio  $a_f/h$  as

$$\frac{a_f}{h} = \frac{0.0024}{0.012} = 0.2$$

Thus  $a_f/h$  varies from near zero to approximately 0.2. From Fig. 5–27, for this range  $\beta$  is nearly constant at approximately 1.05. We will assume it to be so, and re-evaluate  $a_f$  as

$$a_f = \frac{1}{\pi} \left( \frac{81}{1.05(937.5)} \right)^2 = 0.00216 \text{ m}$$

Thus, from Eq. (6–6), the estimated remaining life is

$$\begin{aligned} N_f &= \frac{1}{C} \int_{a_i}^{a_f} \frac{da}{(\beta \Delta \sigma \sqrt{\pi a})^m} = \frac{1}{114 \times 10^{-15}} \int_{0.0001}^{0.00216} \frac{da}{[1.05(937.5) \sqrt{\pi a}]^3} \\ &= -\frac{825.8}{\sqrt{a}} \bigg|_{0.0001}^{0.00216} = 64.8(10^3) \text{ cycles} \end{aligned}$$