King Saud University

Department of Mathematics

Final Exam **205-Math** **2 Semester (1439/1440)**

Question1(4°). Find a point Q on the surface S: $z = 3x^2 - x + y^2 - \frac{1}{36}$ at witch the normal line L is perpendicular to the plane P: $\frac{x}{2} - \frac{y}{3} - z - \frac{1}{36} = 0$.

Question2 (4°). Find the point(s) on the curve $x^2 y = 54$ nearest the origin.

Question3 (5°). (a) Study the continuity of the function $f(x,y) = \begin{cases} \frac{5x^2 y - 3y^3}{x^2 + y^2} & , \quad (x,y) \neq (0,0) \\ 0 & , \quad (x,y) = (0,0) \end{cases}$ at the origin and then: (b) Find the derivatives of the above function with respect to x and with respect to y at the origin.

Question4 (5°). Find the value of $\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}$ at the point (π, π, π) if z is defined as a function of x and y by the equation: $\sin(x + y) + \sin(y + z) = \cos(x/4 + z/4)$

Question5 (4°). Set up the integral for finding the volume of the solid bounded by the surfaces:

$$x = 0 \,,\; y = 0 \,,\; z = 2 \; and \; 2x + y + z - 6 = 0.$$

Question6 (4°). Find the surface area of the surface $z = y^2$ over the triangle with vertices:

(0,0,0), (0,2,0), (2,2,0).

Question7 (4°). Find $\lim_{n\to\infty} x_n$ if: (a) $x_n = (-1)^n \sqrt[n]{n\, 2^{n+1}}$ (b) $x_n = (\frac{e^n + 1}{3^n}) \cos(\pi n)$

Question8 (4°). Check whether the following series is absolutely convergent, conditionally convergent or divergent: (a) $\sum_{n=1}^{\infty} (-1)^n \sqrt[n]{n\, 2^{n+1}}$ (b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\pi^n}$

Question9 (6°). (a) Find the power series representation for the function $f(x) = \frac{2}{(1+x)^3}$ and write the domain of convergence.

(b) Find the sum of the number series $\sum_{n=2}^{\infty} \frac{(-1)^n n(n-1)}{3^n}$