King Saud University

Department of Mathematics

Final Exam **205-Math** **2 Semester (1439/1440)**

Question1 (4°). Find parametric equations of the tangent line to the curve

$$r(t)=(t^3+3)i+(4-2t^3)j+(t^2-4)k \quad \text{at the point } P(2,6,-3)$$

Question2 (4°). Decide whether the function $f(x,y)=\dfrac{x^4+y^4}{x^2+y^2}$ has a limit at the origin.

Question3 (4°). If $z=f(x,y)$ is defined implicitly as a function of x and y by the equation

$xe^y+ye^z=2+3\ln 2-2\ln x$, then find the gradient of z at the point $(1,\ln 2)$.

Question4 (4°). find the point(s) on the curve $y=x^2-4x+5$ nearest the point $P(2,0)$.

Question5 (4°). Convert the following polar integral to cartesian and find its value:

$$\int_0^{\pi/2}\int_0^1 r^3\sin\theta\cos\theta\, dr d\theta$$

Question6 (4°). Find the surface area of the surface $z=y^2$ over the triangle in *xy*-plane with vertices $(0,0)$, $(0,2)$, $(2,2)$.

Question7 (4°). Find the volume of the solid bounded by the surfaces: $z=\sqrt{x^2+y^2}$ *and* $z=2$.

Question8 (3°). Find $\lim\limits_{n\to\infty} x_n$ if: (a) $x_n=\dfrac{(-1)^n\sqrt{n}\,\sin\sqrt{n}}{n+1}$ (b) $x_n=\dfrac{(-1)^n+\sqrt[n]{4^{n^2}n}}{4^n}$

Question9 (4°). Check whether the following series is absolutely convergent, conditionally

convergent or divergent: (a) $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{\sqrt[3]{n}}{n+1}$ (b) $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{e^n}{n!}$ (c) $\sum\limits_{n=1}^{\infty}\dfrac{e^n}{1+e^{\frac{1}{2}n}}$

Question 10 (5° **).** (a) Find the sum of the function series $\sum\limits_{n=0}^{\infty}\dfrac{(-1)^n x^{n+1}}{n+1}$ and its radius and interval of convergence. (b) Find the sum of the number series $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{n\,2^n}$