Chapter 2:

Page 91
3.2 An overseas shipment of 5 foreign automobiles contains 2 that have slight paint blemishes. If an agency receives 3 of these automobiles at random, list the elements of the sample space S, using the letters B and N for blemished and nonblemished, respectively; then to each sample point assign a value x of the random variable X representing the number of automobiles with paint blemishes purchased by the agency.

Solution:

Sample Space	X
NNN	0
NNB	1
NBN	1
BNN	1
BBN	2
BNB	2
NBB	2

3.5 Determine the value c so that each of the following functions can serve as a probability distribution of the discrete random variable X:
(a) $f(x) = c(x^2 + 4),\ for\ x = 0, 1, 2, 3.$
(b) $f(x) = c\binom{2}{x}\binom{3}{3-x},\ for\ x = 0, 1, 2.$

Solution:

(a) $\sum_{x=0}^{3} f(x) = 1$, then

$$c(0^2 + 4) + c(1^2 + 4) + c(2^2 + 4) + c(3^2 + 4) = 1$$

$$4c + 5c + 8c + 13c = 30c \Rightarrow c = \frac{1}{30}$$

(b) $\sum_{x=0}^{2} f(x) = 1$, then

$$c\binom{2}{0}\binom{3}{3-0} + c\binom{2}{1}\binom{3}{3-1} + c\binom{2}{2}\binom{3}{3-2} = 1$$

$$c + 6c + 3c = 10c \Rightarrow c = \frac{1}{10}$$

3.6 The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$f(x) = \begin{cases} \dfrac{20{,}000}{(x+100)^3}, & x > 0, \\ 0, & elsewhere. \end{cases}$$

Find the probability that a bottle of this medicine will have a shell life of
(a) at least 200 days;
(b) anywhere from 80 to 120 days.

Solution:

$$P(X > 200) = 1 - P(X < 200) = 1 - \int_0^{200} \frac{20{,}000}{(x+100)^3} dx$$

$$= 1 - 20{,}000 \int_0^{200} (x+100)^{-3} dx$$

$$= 1 - 20{,}000 \frac{(x+100)^{-2}}{-2} \bigg|_0^{200} = 1 + 10{,}000 \left(\frac{1}{300^2} - \frac{1}{100^2} \right)$$

$$= \frac{1}{9}$$

$$P(80 < X < 120) = \int_{80}^{120} \frac{20{,}000}{(x+100)^3} dx = 20{,}000 \frac{(x+100)^{-2}}{-2} \bigg|_{80}^{120}$$

$$= -10{,}000 \left(\frac{1}{220^2} - \frac{1}{180^2} \right) = 0.10203$$

3.14 The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ 1 - e^{-8x}, & x \geq 0. \end{cases}$$

Find the probability of waiting less than 12 minutes between successive speeders
(a) using the cumulative distribution function of X;
(b) using the probability density function of X.

Solution:

(a) $P(X < 0.2) = F(0.2) = 0.7981$

(b) $f(x) = \frac{d}{dx}F(x) = 8e^{-8x},\ x \geq 0$

$$P(X < 0.2) = \int_0^{0.2} 8e^{-8x}dx = -e^{-8x}|_0^{0.2} = 0.7981$$

3.21 Consider the density function

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1, \\ 0, & elsewhere. \end{cases}$$

(a) Evaluate k.

(b) Find F(x) and use it to evaluate P(0.3 < X < 0.6).

Solution:

(a) $\int_0^1 k\sqrt{x}dx = k\frac{x^{3/2}}{3/2}\Big|_0^1 = \frac{2}{3}k$, then $k = \frac{3}{2}$.

(b) $F(x) = \begin{cases} \int_0^x \frac{3}{2}\sqrt{x}dx = x^{3/2}, & 0 < x < 1 \\ 0, & elsewhere. \end{cases}$

$$F(x) = \begin{cases} x^{3/2}, & 0 < x < 1 \\ 0, & elsewhere. \end{cases}$$

$$P(0.3 < X < 0.6) = F(0.6) - F(0.3) = 0.6^{\frac{3}{2}} - 0.3^{\frac{3}{2}} = 0.300$$

H.W: 3.9- 3.12- 3.26

Chapter 3:

4.12 If a dealer's profit, in units of $5000, on a new automobile can be looked upon as a random variable X having the density function

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & elsewhere. \end{cases}$$

find the average profit per automobile.

Solution:

$$E(X) = \int_0^1 2x(1-x)dx = 2\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Bigg|_0^1 = \frac{1}{3}$$

4.17 Let X be a random variable with the following probability distribution:

x	-3	6	9
f(x)	1/6	1/2	1/3

Find $\mu_{g(X)}$, where $g(X) = (2X + 1)^2$.

Solution:

$$\mu_{g(X)} = E[g(X)] = E[(2X + 1)^2] = E(4X^2 + 4X + 1)$$
$$= 4E(X^2) + 4E(X) + 1$$

x	-3	6	9	$\sum$
$f(x)$	1/6	1/2	1/3	1
$xf(x)$	-1/2	3	3	11/2
$x^2f(x)$	3/2	18	27	93/2

$$\mu_{g(X)} = 4\frac{93}{2} + 4\frac{11}{2} + 1 = 209$$

4.32 In Exercise 3.13 on page 92, the distribution of the number of imperfections per 10 meters of synthetic fabric is given by

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

(b) Find the expected number of imperfections, E(X) = μ.
(c) Find $E(X^2)$.

Solution:

x	0	1	2	3	4	$\sum$
$f(x)$	0.41	0.37	0.16	0.05	0.01	1
$xf(x)$	0	0.37	0.32	0.15	0.04	0.88
$x^2f(x)$	0	0.37	0.64	0.45	0.16	1.62

$$E(X) = 0.88, \ \ E(X^2) = 1.62$$

4.43 The length of time, in minutes, for an airplane to obtain clearance for takeoff at a certain airport is a random variable Y = 3X −2, where X has the density function

$$f(x) = \begin{cases} \frac{1}{4}e^{-x/4}, & x > 0, \\ 0, & elsewhere. \end{cases}$$

Find the mean and variance of random variable Y.

$$E(Y) = E(3X - 2) = 3E(X) - 2$$

$$V(Y) = V(3X - 2) = 9V(X)$$

$$E(X) = \int_0^\infty \frac{x}{4}e^{-x/4}dx = 4$$

$$let\ u = x, dv = e^{-x/4}, then\ v = -4e^{-\frac{x}{4}}, and\ du = 1$$

$$uv - \int vdu = xe^{-\frac{x}{4}} - \int -4e^{-\frac{x}{4}}dx = xe^{-\frac{x}{4}} - 16e^{-\frac{x}{4}}$$

$$\int_0^\infty \frac{x}{4}e^{-x/4}dx = \frac{1}{4}\left(xe^{-\frac{x}{4}} - 16e^{-\frac{x}{4}}\right)\Big|_0^\infty = \frac{1}{4}(0 - 0 - 0 + 16) = 4$$

$$E(X^2) = \int_0^\infty \frac{x^2}{4}e^{-x/4}dx = 32$$

$$let\ u = x^2, dv = e^{-x/4}, then\ v = -4e^{-\frac{x}{4}}, and\ du = 2x$$

$$uv - \int vdu = x^2e^{-\frac{x}{4}} - 2\int -4xe^{-\frac{x}{4}}dx = x^2e^{-\frac{x}{4}} + 8\int xe^{-\frac{x}{4}}dx$$

$$= x^2e^{-\frac{x}{4}} + 8\left(xe^{-\frac{x}{4}} - 16e^{-\frac{x}{4}}\right)$$

$$\int_0^\infty \frac{x^2}{4} e^{-x/4} dx = \frac{1}{4}\left(x^2 e^{-\frac{x}{4}} + 8\left(xe^{-\frac{x}{4}} - 16e^{-\frac{x}{4}}\right)\right)\Bigg|_0^\infty = \frac{1}{4}(16 * 8) = 32$$

$$V(X) = E(X^2) - [E(X)]^2 = 32 - 16 = 16$$

$$E(Y) = 3E(X) - 2 = 3 * 4 - 2 = 10$$

$$V(Y) = 9V(X) = 9 * 16 = 144$$

4.49 Consider the situation in Exercise 4.32 on page119. The distribution of the number of imperfections per 10 meters of synthetic failure is given by

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

Find the variance and standard deviation of the number of imperfections.

Solution:

x	0	1	2	3	4	$\sum$
$f(x)$	0.41	0.37	0.16	0.05	0.01	1
$xf(x)$	0	0.37	0.32	0.15	0.04	0.88
$x^2 f(x)$	0	0.37	0.64	0.45	0.16	1.62

$$E(X) = 0.88,\ \ E(X^2) = 1.62$$

$$V(X) = E(X^2) - [E(X)]^2 = 0.8456$$

$$\sigma = \sqrt{V(X)} = 0.919565$$

H.W: 4.22- 4.28- 4.40- 4.50

(من ملزمة التمارين)

4.1. DISCRETE DISTRIBUTIONS

Q7 & Q12

4.2. CONTINUOUS DISTRIBUTIONS

Q1

4.3. CHEBYSHEV'S THEOREM :(من ملزمة التمارين)

Q1 & Q3 & Q4 & Q5

H.W : Q2