Chapter (5)
Discrete Probability Distributions
Examples

Example (1)

Two balanced dice are rolled. Let X be the sum of the two dice. Obtain the probability distribution of X.

Solution

When the two balanced dice are rolled, there are 36 equally likely possible outcomes as shown below:

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

- The possible values of X are: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.
- The discrete probability distribution of X is given by

X	P(X)
2	1 / 36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5 / 36
9	4/36
10	3/36
11	2/36
12	1 / 36
Total	36/36 =1

Example (2)

The number of persons *X*, in Al Riyadh family chosen at random has the following probability distribution:

X	1	2	3	4	5	6	7	8	Total
P(X)	0.34	0.44	0.11	0.06	0.02	0.01	0.01	0.01	1

1/ Find the average family size {E(X)}

2/ Find the variance of probability distribution

Solution

X	P(X)	XP(X)	$(X-\mu)$	$(X-\mu)^2$	$P(X)*(X-\mu)^2$
1	0.34	0.34	-1.1	1.21	0.4114
2	0.44	0.88	-0.1	0.01	0.0044
3	0.11	0.33	0.9	0.81	0.0891
4	0.06	0.24	1.9	3.61	0.2166
5	0.02	0.1	2.9	8.41	0.1682
6	0.01	0.06	3.9	15.21	0.1521
7	0.01	0.07	4.9	24.01	0.2401
8	0.01	0.08	5.9	34.81	0.3481
		2.1			1.63

$\mu = x\,P(x) = E(x)$ = 2.1

$\sigma^2 = (x-\mu)^2 P(x)$ = 1.63

Example (3)

John Ragsdale sells new cars for Pelican Ford. John usually sells the largest number of cars on Saturday. He has developed the following probability distribution for the number of cars he expects to sell on a particular Saturday.

x	0	1	2	3	4	**Total**
$p(x)$	0.10	0.20	0.30	0.30	0.10	$\sum p(x) = 1$

Find:

1. Expected value of x (The mean of probability distribution)
2. σ^2 (The variance of probability distribution)

Solution:

$\mu = x\ P(x) = E(x) = 2.1$

$\sigma^2 = (x-\mu)^2 P(x) = 1.29$

x	$p(x)$	$x\ P(x)$	$(x-\mu)$	$(x-\mu)^2$	$(x-\mu)^2\ p(x)$
0	0.10	0	-2.1	4.41	0.441
1	0.20	0.20	-1.1	1.21	0.242
2	0.30	0.60	-0.1	0.01	0.003
3	0.30	0.90	0.9	0.81	0.243
4	0.10	0.40	1.9	3.61	0.361
	1	2.1			1.29

Example (4)

Which one of these tables is actually a probability distribution?

x	P(x)
5	0.3
10	0.3
15	0.2
20	0.4

A

x	P(x)
5	0.1
10	0.3
15	0.2
20	0.4

B

x	P(x)
5	0.5
10	0.3
15	-0.2
20	0.4

C

Binomial distribution

(Binomial Probability Distribution):

$$P(X=x) = {}_nC_x\,(\pi)^x(1-\pi)^{(n-x)}$$

$$\mu = n\pi\ ,\ \sigma^2 = n\pi(1-\pi),\ \sigma = \sqrt{n\pi(1-\pi)}$$

Example (5)

If the experiment is tossing a coin 6 times, what is the probability of:

1. Getting two heads.
2. Getting at least 4 heads.
3. Getting at most one head.
4. Getting at least two heads
5. Find mean, variance, and deviation

Solution:

1. $P(X=2) = {}_6C_2(0.5)^2(0.5)^{(6-2)} = 0.2344$
2. $P(X\geq 4) = P(X=4) + P(X=5) + P(X=6) = 0.3434$
3. $P(X\leq 1) = P(X=0) + P(X=1)$
   $= {}_6C_0(0.5)^0(0.5)^{(6-0)} + {}_6C_1(0.5)^1(0.5)^{(6-1)}$
   $= 0.0156 + 0.0938 = 0.1094$
4. $P(X\geq 2) = P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$
   $= 1 - P(X\leq 1)$
   $= 1 - 0.1094 = 0.8906$
5. $\mu = 6(0.5) = 3$
   $\sigma^2 = 6(0.5)(1-0.5) = 1.5$
   $\sigma = \sqrt{1.5} = 1.22$

Example (6)

Over a long period of time it has been observed that a given marksman can hit a target on a single trial with probability equal to 0.8. Suppose he fires four shots at the target. Answer the following:

1. What is the probability that he will hit the target exactly two times?
2. What is the probability that he will hit the target at least once?
3. Find mean, variance, and standard deviation.

Solution:

1. $P(X=2) = {}_4C_2(0.8)^2(0.2)^{(4-2)} = 0.1536$
2. $P(X\geq1) = 1-[P(X=0)]$
   $= 1-[{}_4C_0(0.8)^0(0.2)^{(4-0)}] = 1-0.0016 = 0.9984$
3. $\mu = 4(0.8) = 3.2$
   $\sigma^2 = 4(0.8)(1-0.8) = 0.64$
   $\sigma = \sqrt{0.64} = 0.8$

Example (7)

Find the probability of guessing correctly exactly 6 of the 10 answers on a true-false examination.

Solution:

$P(X=6) = {}_{10}C_6(0.5)^6(0.5)^{(10-6)} = 0.2051$

Poisson distribution

(Poisson Probability Distribution):

$$P(X=x) = \frac{e^{-\lambda}\,\lambda^{x}}{x!}$$

$$\mu = \lambda\ ,\ \sigma^2 = \lambda,\ \sigma = \sqrt{\lambda}$$

Example (8)

The average number of traffic accidents on a certain section of highway is two per week assume that the number of accidents follows a Poisson distribution.

1. Find the probability of no accidents on this section of highway during a 1-week period.
2. Find the probability of at most three accidents on this section of highway during a 1-week.
3. Find the probability of at least four accidents during a 1-week
4. Find variance and standard deviation.

Solution:

1. $P(X=0) = \frac{e^{-2}\,2^{0}}{0!} = \frac{e^{-2}}{1} = 0.1353$
2. $P(X\leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

   $= \frac{e^{-2}\,2^{0}}{0!} + \frac{e^{-2}\,2^{1}}{1!} + \frac{e^{-2}\,2^{2}}{2!} + \frac{e^{-2}\,2^{3}}{3!}$

   $= 0.1353 + 0.2707 + 0.2707 + 0.1804 = 0.8571$
3. $P(X\geq 4) = 1-[P(X\leq 3)]$ note:(the only way is using complement)

   $= 1-[0.8571] = 0.1429$
4. $\mu = 2$

   $\sigma^2 = 2$

   $\sigma = \sqrt{2} = 1.41$

Example (9)

The number of people arriving to a movie theater in a 30-minute time period is best modeled using which of the following distributions?

A) Normal	B) Binomial	C) Hypergeometric	D) Poisson

Example (10)

A manufacturing process produces defective items 5% of the time. A sample of 20 items is taken for quality control. If you wanted to determine the probability that exactly 2 of the 20 items are defective, which distribution should be used?

A) Uniform	B) Binomial	C) Hypergeometric	D) Poisson

Example (11)

Which of the following is not a requirement of a binomial distribution?

A) A constant probability of success.
B) Only two possible outcomes.
C) A fixed number of trials.
D) Equally likely outcomes

Chapter (6)
Continuous Probability Distributions
Normal probability distribution
Examples

Example (1)
Let X be a normally distributed random variable with mean 65 and standard deviation 13. Find the standard normal random variable (z) for P(X>80)

Solution:

$P(X>80) = P(\frac{X-\mu}{\sigma}) > \frac{80-65}{13})$
$= P(Z >1.15)$
$=1- P(Z <1.15) =1-0.8749$

Example (2)
If the mean = 65 and standard deviation =13. Find x from the following:
1. z = 0.6
2. z = -1.93

Solution:

$Z=\frac{X-\mu}{\sigma} \Longleftrightarrow X=\mu+\sigma Z$

1. X=65+13(0.6) = 72.8
2. X=65+13(-1.93) = 39.91

Example (3) "The Empirical Rule"
A sample of the rental rates at University Park Apartments approximates a systematical, bell- shaped distribution. The sample mean is \$500; the standard deviation is \$20.Using the Empirical Rule, answer these questions:

1. About 68.26 percent of the monthly food expenditures are between what two amounts?
2. About 95.44 percent of the monthly food expenditures are between what two amounts?

3. About all the monthly (99.73%) food expenditures are between what two amounts?

Solution:

a. $\mu \pm \sigma = 500 \pm 20 = (480, 520)$
b. $\mu \pm 2\sigma = 500 \pm 40 = (460, 540)$
c. $\mu \pm 3\sigma = 500 \pm 60 = (440, 560)$

Example (4)
The mean of a normal probability distribution is 120; the standard deviation is 10.

a. About 68.26 percent of the observations lie between what two values?
b. About 95.44 percent of the observations lie between what two values?
c. About 99.73 percent of the observations lie between what two values?

Solution:

a. $\mu \pm \sigma = 120 \pm 10 = (110, 130)$
b. $\mu \pm 2\sigma = 120 \pm 20 = (100, 140)$
c. $\mu \pm 3\sigma = 120 \pm 30 = (90, 150)$

Example (5)

Studies show that gasoline use for compact cars sold in the United States is normally distributed, with a mean of 25.5 miles per gallon (mpg) and a standard deviation of 4.5 mpg. Find the probability of compact cars that get:

1. 30 mpg or more.
2. 30 mpg or less.
3. Between 30 and 35.
4. Between 30 and 21.

Solution:

1. $P(X \geq 30) = P(\frac{X-\mu}{\sigma} \geq \frac{30-25.5}{4.5})$

   $= P(Z \geq 1)$

   $= 1 - P(Z < 1) = 1 - 0.8413 = 0.1587$
2. $P(X \leq 30) = 1 - P(X \geq 30) = 0.8413$
3. $P(30 \leq X \leq 35) = P(\frac{30-25.5}{4.5} \leq \frac{X-\mu}{\sigma} \leq \frac{35-25.5}{4.5})$

   $= P(1 \leq Z \leq 2.11)$

   $= P(Z \leq 2.11) - P(Z \leq 1)$

   $= 0.9826 – 0.8413 = 0.1413$

4. $P(21 \leq X \leq 30) = P(\frac{21-25.5}{4.5} \leq \frac{X-\mu}{\sigma} \leq \frac{30-25.5}{4.5})$

   $= P(-1 \leq Z \leq 1)$

   $= P(Z \leq 1) - P(Z \leq -1)$

   $= 0.8413 – 0.1587 = 0.6826$

Example (6)

Suppose that X ~N (3, 0.16). Find the following probability:

1. $P(x \geq 3)$.

2. $P(2.8 < x < 3.1)$.

Solution:

1. 0.5
2. 0.6284

Example (7)

The grades on a short quiz in math were 0, 1, 2…, 10 point, depending on the number answered correctly out of 10 questions. The mean grade was 6.7 and the standard deviation was 1.2. Assuming the grades to be normally distributed, determine:

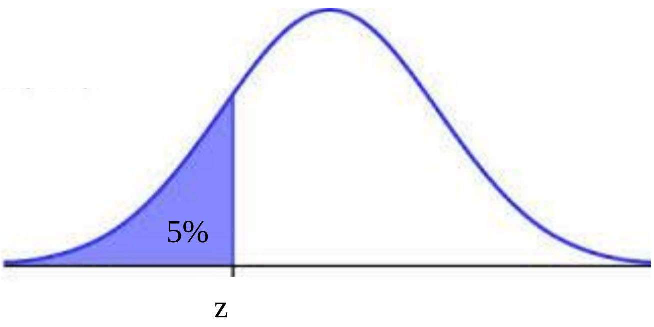
1. The percentage of students scoring more than 6 points.
2. The percentage of students scoring less than 8 points.
3. The percentage of students scoring between 5.5and 6 points.
4. The percentage of students scoring between 5.5and 8 points.
5. The percentage of students scoring less than 5.5 points.
6. The percentage of students scoring more than 8 points.
7. The percentage of students scoring equal to 8 points.
8. The maximum grade of the lowest 5 % of the class.
9. The minimum grade of the highest 15 % of the class.

Solution:

1. 0.7202(100) = 72.02%
2. 0.8607(100) = 86.07%
3. 0.1212(100) =12.12%%
4. 0.7020(100) =70.20%
5. 0.1586(100) =15.86%
6. 0.1393(100) =13.93%
7. 0(100) = 0%

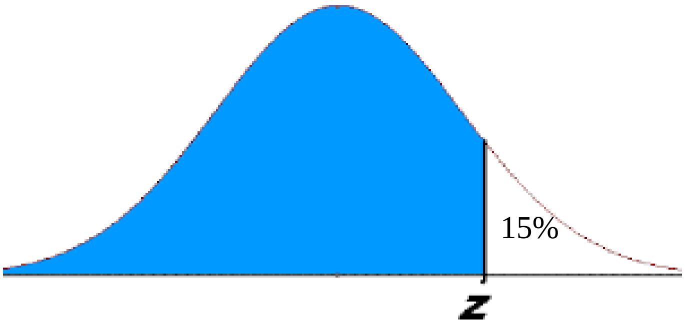
8. $P(Z \leq z)= 0.05$
   $Z= -2.58$

$X=\mu+\sigma Z = 6.7-1.2(2.58) =3.6$ grade



9. $P(Z \leq z)= 0.85$
   $Z= 1.04$

$X=\mu+\sigma Z = 6.7+1.2(1.04) =7.9$ grade



Example (8)

If the heights of 300 students are normally distributed, with a mean 172 centimeters and a standard deviation 8 centimeters, how many students have heights?

1. Greater than 184 centimeters.
2. Less than or equal to 160 centimeters.
3. Between 164 and 180 centimeters inclusive.
4. Equal to 172 centimeters.

Solution:

1. 0.0668(300) = 20.04=20 students
2. 0.0668(300) = 20.04=20 students
3. 0.6827(300) = 204.81=205 students
4. 0

Example (9)

What is the area under a normal curve that falls between the mean and one standard deviation below the mean?

Solution:

Using empirical role:

$(\frac{1}{2})(0.6826) = 0.3413$