MATHEMATICS

Expand function in Legendre polynomials on the interval [-1,1]

Asked 1 year, 11 months ago Active 26 days ago Viewed 2k times

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Expand the following function in Legendre polynomials on the interval [-1,1] :

$$f(x) = |x|$$

The Legendre polynomials $p_n(x)$ are defined by the formula :

$$p_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^2$$

for $n = 0, 1, 2, 3, \ldots$

My attempt :

we have using the fact that $|x|$ is an even function.

$$a_0 = \frac{2}{\pi}$$

$$a_n = \frac{2}{\pi}\int_{-1}^{1} x\cos(nx)\,dx$$

Then what is the next step ?

special-functions orthogonal-polynomials legendre-polynomials

edited Nov 29 '17 at 8:55
PM. 3,928 2 9 25

asked Nov 22 '17 at 13:03
user326307 285 2 11

You have edited this, removed your original question and turned it in to a brand new question. This doesn't work well with existing answers to your original question. It would be better to add an entirely new question. – PM. Nov 28 '17 at 11:35

2 Answers

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$$f(x) = \sum_{n=0}^{\infty} a_n P_n(x)$$

See for example here for more information.

Another very useful reference (in general) is Abramowitz and Stegun's Handbook of Mathematical Functions where you will find chapters on Legendre functions and orthogonal polynomials.

edited Nov 22 '17 at 14:42

answered Nov 22 '17 at 14:06

PM.
3,928 2 9 25

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We know that the Fourier-Legendre series is like

$$f(x) = \sum_{n=0}^{\infty} C_n P_n(x)$$

where

$$C_n = \frac{2n+1}{2} \int_{-1}^{1} f(x) P_n(x)\, dx$$

So now we are going to calculate the result of

$$\frac{2n+1}{2} \int_{-1}^{1} |x| P_n(x)\, dx$$

As $|x|$ is an even function, and the parity of $P_n(x)$ depends on the parity of n, We can write

$$\int_{-1}^{1} |x| P_n(x)\, dx$$

as

$$\int_{-1}^{1} |x| P_{2k}(x)\, dx \;\; k = 0, 1, 2...$$

and

$$\int_{-1}^{1} |x| P_{2k}(x)\, dx$$

$$= \int_{-1}^{0} -x P_{2k}(x)\, dx + \int_{0}^{1} x P_{2k}(x)\, dx$$

$$= 2 \int_{0}^{1} x P_{2k}(x)\, dx$$

we get

$$2\int_0^1 xP_{2k}(x)\,dx$$
$$=2(\frac{2k+1}{4k+1}\int_0^1 P_{2k+1}(x)\,dx+\frac{2k}{4k+1}\int_0^1 P_{2k-1}(x)\,dx)$$

As

$$\int_0^1 P_n(x)\,dx=\begin{cases}0 & n=2k\\ \frac{(-1)^k(2k-1)!!}{(2k+2)!!} & n=2k+1\end{cases}$$

We get

$$C_{2k}=(2k+1)\frac{(-1)^k(2k-1)!!}{(2k+2)!!}+n\frac{(-1)^{k-1}(2k-3)!!}{(2k)!!}$$
$$=\begin{cases}\frac{1}{2} & k=0\\ \frac{(-1)^{k+1}(4k+1)}{2^{2k}(k-1)!}\frac{(2k-2)!}{(k+1)!} & k>0\end{cases}$$

So

$$|x|=\frac{1}{2}+\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(4k+1)}{2^{2k}(k-1)!}\frac{(2k-2)!}{(k+1)!}P_{2k}(x)$$

answered May 18 at 14:18

Steven Yang
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