

Exercises -1-

1.1) A diet conscious housewife wishes to ensure certain minimum intake of vitamins A, B and C for the family. The minimum daily (quantity) needs of vitamins A,B and C for the family are respectively 30, 20 and 16 for the supply of theses minimum vitamin requirements, the house wife relies on two fresh foods. The **first** one provides 7, 5, 2 units of the three vitamins per gram respectively and the **second** one provides 2, 4, 8 units of the same three vitamins per gram of the foodstuff respectively. The first foodstuff costs 3\$ per gram and the second 2\$ per gram. **The problem is how many grams of each foodstuff should the housewife buy every day to keep her food bills as low as possible?**
(Formulate the problem as liner programming problem.)

Answer:

let x_1 : the number of units of food 1.

let x_2 : the number of units of foods 2.

The data of the given problem can be summarized as below:

Food	Content of vitamins type			Cost per unit (\$)
	A	B	C	
x_1	7	5	2	3
x_2	2	4	8	2
Minimum vitamins required	30	20	16	

Objective function:

Minimum $Z = 3x_1 + 2x_2$

subject to the Constraints:

$$(1) \quad 7x_1 + 2x_2 \geq 30$$

$$(2) \quad 5x_1 + 4x_2 \geq 20$$

$$(3) \quad 2x_1 + 8x_2 \geq 16$$

$$(4) \quad x_1, x_2 \geq 0$$

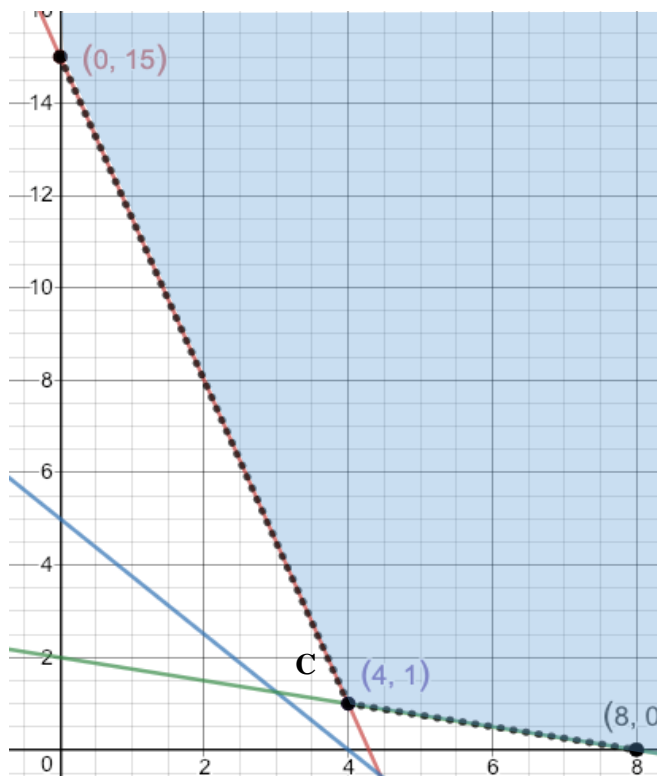
- Solve graphically a Linear Programming model that will allow the housewife to minimize the cost. And determine the optimal solution.

To determine two points on the Constraints as follow

$$7x_1 + 2x_2 = 30 \gg (0, 15) \text{ and } (4.3, 0)$$

$$5x_1 + 4x_2 = 20 \gg (0, 5) \text{ and } (4, 0)$$

$$2x_1 + 8x_2 = 16 \gg (0, 2) \text{ and } (8, 0)$$



The optimal solution of an LPP occurs at point C. The values of associated with the optimum point C are determined by solving the equations associated with lines (1) and (3), that is,

$$4* (7x_1 + 2x_2 = 30)$$

$$(-)* (2x_1 + 8x_2 = 16)$$

$$(28 - 2)x_1 = 120 - 16$$

$$26x_1 = 104 \gggg x_1^* = 4$$

$$x_2^* = \frac{30-7(4)}{2} = 1$$

Or we can find the optimal solution by compute the value of the objective function at each vertex, as follows

Points (x_1, x_2)	$Z = 3x_1 + 2x_2$
(0,15)	30
(4,1)	14
(8,0)	24

1.2) The manager of an oil refinery has to decide upon the optimal mix of two possible blending processes of which the inputs and outputs per production run are as follows

Process	input		Output	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	5	3	5	8
2	4	5	4	4

The maximum amount available of crude A and B are 200 units and 150 units respectively. Market requirements show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The **profits** per production run from process 1 and process 2 are 3\$ and 4\$ respectively.

Formulate the problem as liner programming problem.

Answer:

let x_1 : the number of production from processes 1.

let x_2 : the number of production from processes 2.

Objective function:

Maximize $Z = 3x_1 + 4x_2$

Constrains:

$$(1) 5x_1 + 4x_2 \leq 200$$

$$(2) 3x_1 + 5x_2 \leq 150$$

$$(3) 5x_1 + 4x_2 \geq 100$$

$$(4) 8x_1 + 4x_2 \geq 80$$

$$(5) x_1, x_2 \geq 0$$

1.3) A workshop has three (3) types of machines A, B and C; it can manufacture two (2) products 1 and 2, and all products have to go to each machine and each one goes in the same order; First to the machine A, then to B and then to C. The following table shows:

- The hours needed at each machine, per product unit.
- The total available hours for each machine, per week.
- The profit of each product per unit sold.

Type of Machine	Product 1	Product 2	Available hours per week
A	2	2	16
B	1	2	12
C	4	2	28
Profit per unit	1	1.50	

Formulate and solve using the **graphical method** a **Linear Programming model** for the previous situation that allows the workshop to obtain maximum gains.

Answer:

let x_1 : the number of product 1 per week .

let x_2 : the number of product 2 per week .

Objective function:

Maximize $Z = x_1 + 1.5x_2$

Constraints:

(1) $2x_1 + 2x_2 \leq 16$

(2) $x_1 + 2x_2 \leq 12$

(3) $4x_1 + 2x_2 \leq 28$

(4) $x_1, x_2 \geq 0$

For the graphical solution:

Step 1: since $x_1, x_2 \geq 0$, we consider only the first quadrant of x y-plane.

Step 2: we draw straight lines for the equations

$$2x_1 + 2x_2 = 16$$

$$x_1 + 2x_2 = 12$$

$$4x_1 + 2x_2 = 28$$

To determine two point on the straight line $2x_1 + 2x_2 = 16$

put $x_2 = 0 \Rightarrow x_1 = 8$, (8,0) is a point on the line 1.

put $x_1 = 0 \Rightarrow x_2 = 8$, (0,8) is a point on the line 1.

To determine two point on the straight line $x_1 + 2x_2 = 12$

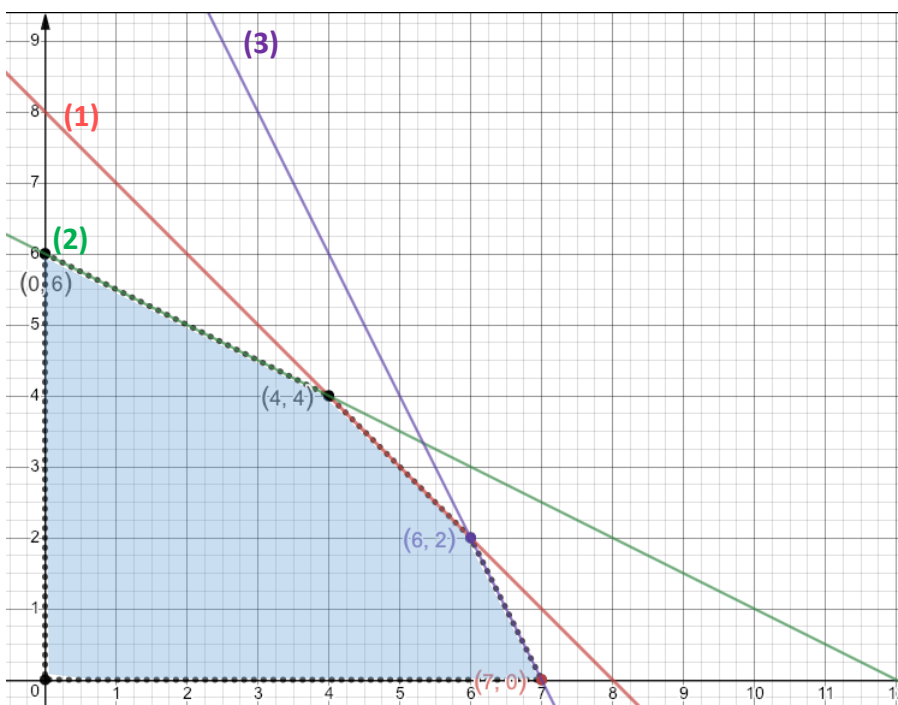
put $x_2 = 0 \Rightarrow x_1 = 12$, (12,0) is a point on the line 2.

put $x_1 = 0 \Rightarrow x_2 = 6$, (0,6) is a point on the line 2.

To determine two point on the straight line $4x_1 + 2x_2 \leq 28$

put $x_2 = 0 \Rightarrow x_1 = 7$, (7,0) is a point on the line 3.

put $x_1 = 0 \Rightarrow x_2 = 14$, (0,14) is a point on the line 3.



The intersection of region is the feasible solution of LPP (linear programming problem).

Therefore, every point in the shaded region is a feasible solution of LPP, since this point satisfies all the constraints including the non-negative constraints.

Technique to find the optimal solution of an LPP

Step 1: Find the coordinates of each vertex (corner point) of the the feasible region. These coordinates can be obtained from the graph *or* by solving the equation of the lines.

Step 2: at each vertex compute the value of objective function.

Step 3: Identify the vertex at which the value of objective function is maximum.

Points (x_1, x_2)	$Z = x_1 + 1.5x_2$
(0,0)	0
(0,6)	0.25
(4,4)	10
(6,2)	9
(7,0)	7

The optimal solution at $x_1 = 4$, $x_2 = 4$ with an optimal value $Z=10$ that represents the workshop's profit.

#Coordinates can be obtained from by solving the equation of the lines, as following

$$2x_1 + 2x_2 = 16$$

$$(-)* (x_1 + 2x_2 = 12)$$

$$(2x_1 - x_1) + (2x_2 - 2x_2) = 16 - 12$$

$$x_1 = 4$$

$$2(4) + 2x_2 = 16$$

$$x_2 = \frac{16-8}{2} = 4$$

$$>> (4,4)$$

$$2x_1 + 2x_2 = 16$$

$$(-)* (4x_1 + 2x_2 = 28)$$

$$-2x_1 = -12$$

$$x_1 = 6$$

$$x_2 = \frac{16 - 12}{2} = 2$$

$$>> (6,2)$$

*Note: we can use the **Graphic Linear Optimizer (GLP)** software to solution like this model.

<https://www.desmos.com/calculator/2rnqgoa6a4>

HW 1.4) A company produces two different products. One of them needs $\frac{1}{4}$ of an hour of assembly work per unit (عمل التجميع), $\frac{1}{8}$ of an hour in quality control work (اعمال ضبط الجودة) and US\$1.2 in raw materials. The other product requires $\frac{1}{3}$ of an hour of assembly work per unit, $\frac{1}{3}$ of an hour in quality control work and US\$0.9 in raw materials. Given the current availability of staff in the company, each day there is at most a total of **90** hours available for **assembly** and **80** hours for **quality control**. The first and second products described have a market value (sale price) of US\$9.0 and \$8.0 per unit respectively. In addition, the maximum amount of daily sales for the first product is estimated to be 200 units, without there being a maximum limit of daily sales for the second product.

Formulate and solve graphically a Linear Programming model that will allow the company to maximize profits.