

Questions on strong mathematical induction

Q1: Let $\{a_n\}$ be a sequence defined inductively as:

$$a_1 = 1, a_2 = 2, a_{n+1} = 2a_n + a_{n-1} \text{ for all } n \geq 2.$$

Prove that $a_n \leq \left(\frac{5}{2}\right)^n$ for all $n \geq 1$.

Q2: Let $\{a_n\}$ be a sequence defined inductively as:

$$a_1 = 1, a_2 = 5, a_{n+1} = 2a_n + 3a_{n-1} \text{ for all } n \geq 2.$$

Prove that $3^n \leq a_{n+1} \leq 2 \times 3^n$ for all $n \geq 1$.

Q3: Let $\{a_n\}$ be a sequence defined inductively as:

$$a_1 = a_2 = a_3 = 1, a_{n+2} = a_{n+1} + a_n + a_{n-1} \text{ for all } n \geq 2.$$

Prove that a_n is an odd number for all $n \geq 1$.

Q4: Let $\{a_n\}$ be a sequence defined inductively as:

$$a_0 = 1, a_{n+1} = a_n + 3^n \text{ for all } n \geq 0.$$

Prove that $a_n = \frac{1}{2}(3^n + 1)$ for all $n \geq 0$.
