Exercise-9-

Given the following Payoff tables : where the table represents profits :

Alternatives	States of nature			
	S1	S2	S3	S4
A1	3	5	8	-1
A2	6	5	2	0
A3	0	5	6	4

State which can be chosen as the best act using:
(a) Maximax, (b) Maximin, (c) Minimax regret (savage criterion),
(d) Equal likelihood (Laplace),
(e) Hurwicz Alpha criterion α=0.4

➢ Decision making under uncertainty.

a) Maximax :

Max(Max Ai) = Max (8, 6, 6) = 8
Decision : Select A1

b) Maxmin :

Max(Min Ai) =Max (-1, 0 ,0) =0
Decision : Select A2 or A3

c) Minimax :

Loss table

Alternatives	States of nature			
	S1	S2	S3	S4
A1	3	0	0	5
A2	0	0	6	4
A3	6	0	2	0

Min(Max Ai)= Min(5, 6, 6) =5
Decision : Select A1

d) Laplace :

we associate equal probability for event say say ¼
Expected pay-offs are :

E(A1)= $\frac{3+5+8-1}{4} = \frac{15}{4} = 3.75$

E(A2)= $\frac{6+5+2+0}{4} = \frac{13}{4} = 3.25$

E(A3)= $\frac{0+5+6+4}{4} = \frac{15}{4} = 3.75$

A1 and A3 has the maximum expected pay-off
Decision : Select A1 or A3

e) Hurwicz :

D= α (Maximum in Ai) + (1 – α)(Minimum in Ai)

D(A1)=(0.4*8)+(0.6*-1) =2.6
D(A2)=(0.4*8)+(0.6*0) =2.4
D(A3)=(0.4*6)+(0.6*0) =2.4

D(A1) has the maximum
Decision : Select A1

Given : P(S1)=0.6 , P(S2)=0.1 , P(S3)=0.2 , P(S4)=0.1 in table (A) ,Find:
(f)Expected Monetary Value (EMV) ,
(g)Expected Opportunity Loss (EOL) ,
(k)Expected Value of Perfect Information (EVPI)

Alternatives	States of nature			
	S1	S2	S3	S4
A1	3	5	8	-1
A2	6	5	2	0
A3	0	5	6	4
probability	0.6	0.1	0.2	0.1

➢ Decision making under Risk.

f) Expected Monetary Value (EMV)

EMV for A1 = (3*0.6)+(5*0.1)+(8*0.2)+(-1*0.1)= 3.8
EMV for A2 = (6*0.6)+(5*0.1)+(2*0.2)+(0*0.1)= 4.5
EMV for A3 = (0*0.6)+(5*0.1)+(6*0.2)+(4*0.1)= 2.1

EMV for A2 is greater
Decision : Select A2

g) Expected Opportunity Loss (EOL) ,

Loss table				
Alternatives	States of nature			
	S1	S2	S3	S4
A1	3	0	0	5
A2	0	0	6	4
A3	6	0	2	0
probability	0.6	0.1	0.2	0.1

ELO for A1 = (3*0.6)+(0*0.1)+(0*0.2)+(5*0.1)= 2.3
ELO for A2 = (0*0.6)+(0*0.1)+(6*0.2)+(4*0.1)= 1.6
ELO for A3 = (6*0.6)+(0*0.1)+(2*0.2)+(0*0.1)= 4

EMV for A2 is least
Decision : Select A2

h) Expected Value of Perfect Information (EVPI)

	Max. profit of each state (S)	probability	Expected value (= prob. * profit)
S1	6	0.6	3.6
S2	5	0.1	0.5
S3	8	0.2	1.6
S4	4	0.1	0.4
Sum *Expected profit with perfect information*			6.1

$EVPI$ = *Expected profit with perfect information* – max EMV

= 6.1 – 4.5
= 1.6

(H.W)

Example: An agricultural company wants to decide which commodity should stock to get maximum profit. It was supplied with the following information. The probability that the monsoon will **be excess, normal and deficient is 0.40,0.30 and 0.30 .** The estimated profit or loss three commodities in respect of these different kinds of monsoon are:

Profit per 1 ton			
Monsoon	Excess	Normal	Deficient
Rice	10,000	-4,000	15,000
Wheat	4,000	-3,000	8,000
Maize	4,000	1,000	-1,000

Determine the optimal decision under each of the following decision criteria and show how you arrived at it:

(a) Maximax, (b) Maximin, (c) Minimax regret (savage criterion), (d) Equal likelihood (Laplace), (e) Hurwicz Alpha criterion α=0.8, (f)EMV, (g)EOL, (k)EVPI .

➢ Decision making under uncertainty.

1. Maximax criterion (OPTIMISM)

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
MAX	**15,000**	**8,000**	**4,000**

The maximum of column maxima is 15,000. Hence, the company should adopt Rice commodity.

2. Maximin criterion (pessimism).

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
Min	**-4,000**	**-3,000**	**-1,000**

The maximum of column minima is -1,000. Hence, the company should adopt Maize commodity.

3. Minimax regret.

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000

Note: From the given payoff matrix, develop an opportunity-loss (or regret) matrix as follows:
(i) Find the best payoff corresponding to each state of nature
(ii) Subtract all other payoff values in that row from this value.

Opportunity Loss Table

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000-10,000= 0	10,000-4,000= 6,000	10,000-4,000= 6,000
Normal	1,000-(-4,000)= 5,000	1,000-(-3,000) = 4,000	1,000-1,000= 0
Deficient	15,000-15,000= 0	15,000-8,000 = 7,000	15,000-(-1,000)= 16,000
MAX	**5,000**	**7,000**	**16,000**

Decision: Rice

4. Equal likelihood (Laplace).

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
Expected (average)	$\frac{1}{3}(10{,}000 - 4{,}000 + 15{,}000) = 7000$	$\frac{1}{3}(4{,}000 - 3{,}000 + 8{,}000) = 3000$	$\frac{1}{3}(4{,}000 - 1{,}000 + 1{,}000) = 1333.33$

Decision: Rice

5. Hurwicz Alpha criterion α=0.8.

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
weighted average: D_i	0.8(15,000)+0.2(-4,000)= 11,200	0.8(8,0000+0.2(-3,000)= 5,800	0.8*4,000+0.2(-1,000)= 3000

$D = \alpha\,(Maximum\ in\ column) + (1 - \alpha)(Minimum\ in\ column)$

Decision: Rice

- **Decision making under Risk.**

1. Expected monetary value (EMV).

Alternative / Monsoon	Rice	Wheat	Maize	probability
Excess	10,000	4,000	4,000	0.4
Normal	-4,000	-3,000	1,000	0.30
Deficient	15,000	8,000	-1,000	0.30
EMV	0.4(10,000)+0.3(-4,000)+0.3(15,000)= 7300	0.4(4,000)+0.3(-3,000)+0.3(8,000)= 3100	0.4(4,000)+0.3(-1,000)+0.3(1,000)= 1600	

Decision: Rice

2. Expected Opportunity Loss (EOL).

Opportunity Loss Table

Alternative / Monsoon	Rice	Wheat	Maize	probability
Excess	0	6,000	6,000	0.4
Normal	5,000	4,000	0	0.30
Deficient	0	7,000	16,000	0.30
EOL	0.3(5,000) =1,500	0.4(6,000)+0.3(4,000)+0.3(7,000) =5,700	0.4(6,000)+0.3(16,000) =7,200	

Decision: Rice

3. Expected Value of Perfect Information (EVPI).

Alternative / Monsoon	Rice	Wheat	Maize	probability
Excess	10,000	4,000	4,000	0.4
Normal	-4,000	-3,000	1,000	0.30
Deficient	15,000	8,000	-1,000	0.30
EMV	0.4(10,000)+0.3(-4,000)+0.3(15,000)= 7300	0.4(4,000)+0.3(-3,000)+0.3(8,000)= 3100	0.4(4,000)+0.3(-1,000)+0.3(1,000)= 1600	

$EVPI = Expected\ profit\ with\ perfect\ information\ - \max EMV$

$= \sum_{i=1}^{m} p_i \max(r_{ij}) - \max EMV.$

$$EVPI = 10{,}000(0.4) + 0.3(1{,}000) + 0.3(15{,}000) - 7{,}300 = 8{,}800 - 7{,}300 = 1{,}500$$