7.
i. JOINT, MARGINAL, AND CONDITIONAL DISTRIBUTIONS
ii. COVARIANCE, CORRELATION, INDEPENDENCE OF VARIABLES (STOCHASTIC INDEPENDENCE)

Q1) The joint probability function of two discrete random variables X and Y is given by f(x,y)= cxy for x=1, 2, 3 and y= 1, 2, 3 and equals zero otherwise. Find:

a. The constant c.
b. P(X=2,Y=3)
c. $P(1 \leq X \leq 2, Y \leq 2)$.
d. $P(X \geq 2)$.
e. $P(Y < 2)$.
f. $P(X = 1)$.
g. $P(Y = 3)$.

Q2) For the random variables of Problem 1, find the marginal probability function of X and Y. Determine whether X and Y are independent.

Q3) Let X and Y be continuous random variables having joint density function

$$f(x,y) = \begin{cases} c(x^2 + y^2) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & otherwise \end{cases}$$

Determine:

a. The constant c.
b. $P\left(X < \frac{1}{2}, Y > \frac{1}{2}\right)$.
c. $P\left(\frac{1}{4} < X < \frac{3}{4}\right)$.
d. $P\left(Y < \frac{1}{2}\right)$.
e. Whether X and Y are independent.

Q4) For the random variables of Problem 3, find the marginal probability function of X and Y.

Q5) For the distribution of Problem 1, find the conditional probability function of X given Y, Y given X.

Q6) Let $f(x,y) = \begin{cases} x+y & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & otherwise \end{cases}$

Find the conditional probability function of X given Y, Y given X.

Q7) For the distribution of Problem 3, find the conditional probability function of X given Y, Y given X.

Q8) Let $f(x,y) = \begin{cases} e^{-(x+y)} & x \ge 0, y \ge 0 \\ 0 & otherwise \end{cases}$

be the joint density function of X and Y. Find the conditional probability function of X given Y, Y given X.

Q9) Let X and Y be random variables having joint density function

$$f(x,y) = \begin{cases} c(2x+y) & 0 < x < 1, 0 < y < 2 \\ 0 & otherwise \end{cases}$$

Find:

a. The constant c.
b. $P\left(X > \frac{1}{2}, Y < \frac{3}{2}\right)$.
c. The (marginal) density function of X.
d. The (marginal) density function of Y.

Q10) The joint probability function for the random variables X and Y is given in following table, then find:

X \ Y	0	1	2
0	1/18	1/9	1/6
1	1/9	1/18	1/9
2	1/6	1/6	1/18

a. The marginal probability functions of X and Y.
b. $P(1 \le X < 3, Y \ge 1)$.

c. Determine whether X and Y are independent.

Q11) Let X and Y be random variables having joint density function

$$f(x,y)=\begin{cases} x+y & 0\le x\le 1, 0\le y\le 1\\ 0 & otherwise\end{cases}$$

Find: a. Var(X). b. Var(Y). c. σ_X. d. σ_Y. e. σ_{XY}. f. ρ.

Q12) Work Problem 11 if the joint density function is

$$f(x,y)=\begin{cases} e^{-(x+y)} & x\ge 0, y\ge 0\\ 0 & otherwise\end{cases}$$

Q13) Find a. The covariance. b. The correlation coefficient of two random variables X and Y. If E(X)=2, E(Y)=3, E(XY)=10, E(X^2)=9, E(Y^2)=16.

Q14) The correlation coefficient of two random variables X and Y is -1/4 while their variances are 3 and 5. Find the covariance.

Q15) Let $f(x,y)=\begin{cases} \frac{xy}{36} & x=1,2,3 \text{ } and \text{ } y=1,2,3\\ 0 & otherwise\end{cases}$

be the joint density function of X and Y. Find the conditional probability function of X given Y, Y given X.

Q16) The joint probability function of two discrete random variables X and Y is given by $f(x,y)=c(2x+y)$, where x and y can assume all integers such that $0\le x\le 2$, $0\le y\le 3$, and $f(x,y)=0$ otherwise. Find:

a. The value of the constant c.
b. $P(X=2,Y=1)$.
c. $P(X\ge 1,Y\le 2)$

Q17) For the Problem 16, find a. E(X). b. E(Y). c. E(XY). d. E(X^2). e. E(Y^2). f. Var(X). g. Var(Y). h. Cov(X,Y). i. ρ.

Q18) The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} 8xy & 0 \le x \le 1, 0 \le y \le x \\ 0 & otherwise \end{cases}$$

Find:

a. The marginal density of X.
b. The marginal density of Y.
c. The conditional density of X.
d. The conditional density of Y.

Q19) Find the conditional expectation of X given Y and Y given X in Problem 18.

Q20) Find the conditional variance of Y given X for Problem 18.