Exercise-6-

The criteria of decision-making under uncertainty is summarized as follows:

(i) To find Optimism (Maximax) criterion.

1. Locate the maximum payoff values corresponding to each decision alternative.
2. Select a decision alternative with best payoff value (maximum for profit).

(ii) Pessimism (Maximin) criterion

1. Locate the minimum payoff value corresponding to each decision alternative.
2. Select a decision alternative with the best payoff value (maximum for profit).

(iii) Equal probabilities (Laplace) criterion.

1. Assign equal probability value to each state of nature by using the formula: *1 ÷ (number of states of nature)*.
2. Compute the expected (or average) payoff for each alternative by adding all the payoffs and dividing by the number of possible states of nature.
3. Select the best expected payoff value (maximum for profit).

(iv) Hurwiez criterion

(i) Decide the coefficient of optimism a (alpha) and then coefficient of pessimism (1 – α).

(ii) For each decision alternative select the largest and lowest payoff value and multiply these with α and (1 – α) values, respectively. Then calculate the weighted average,

$$D = \alpha\,(\text{Maximum in column}) + (1 - \alpha)(\text{Minimum in column})$$

(iii) Select an alternative with best weighted average payoff value.

(v) Minmax (salvage or Regret) criterion

1. from the given payoff matrix, develop an opportunity-loss (or regret) matrix as follows:

- Find the best payoff corresponding to each state of nature
- Subtract all other payoff values in that row from this value.

2. For each decision alternative identify the worst (maximum) payoff value. Record this value in the new row.
3. Select a decision alternative resulting in a smallest anticipated opportunity-loss value.

The criteria of DECISION-MAKING UNDER RISK is summarized as follows:

(i) Expected payoff criterion (E)

1. Construct a payoff matrix listing all possible courses of action and states of nature.
2. Calculate the EMV for each course of action by multiplying the conditional payoffs by the associated probabilities and adding these weighted values for each course of action

$EMV = \sum_{i=1}^{m} r_{ij}\, p_i$

3. Select the course of action that yields the optimal EMV.

(ii) Expected Opportunity Loss (EOL)

1. Prepare a conditional payoff values matrix for each combination of course of action and state of nature along with the associated probabilities.
2. For each state of nature calculate the conditional opportunity loss (COL) values by subtracting each payoff from the maximum payoff.
3. Calculate the EOL for each course of action by multiplying the probability of each state of nature with the COL value and then adding the values.

$EMV = \sum_{i=1}^{m} L_{ij}\, p_i$, L_{ij}: opportunity loss due to state of nature i and course of action j

4. Select a course of action for which the EOL is minimum**.**

(iii) Most likelihood criterion (ML)

ML= choose high probability with highest alternative payoff OR with low alternative cost.

Given the following Payoff tables : where the table represents profits :

Alternatives	States of nature			
	S1	S2	S3	S4
A1	3	5	8	-1
A2	6	5	2	0
A3	0	5	6	4

State which can be chosen as the best act using:
(a) Maximax, (b) Maximin, (c) Minimax regret (savage criterion),
(d) Equal likelihood (Laplace),
(e) Hurwicz Alpha criterion α=0.4

➢ Decision making under uncertainty.

a) Maximax :

Max(Max Ai) = Max (8, 6, 6) = 8
Decision : Select A1

b) Maxmin :

Max(Min Ai) =Max (-1, 0 ,0) =0
Decision : Select A2 or A3

c) Minimax :

Loss table

Alternatives	States of nature			
	S1	S2	S3	S4
A1	3	0	0	5
A2	0	0	6	4
A3	6	0	2	0

Min(Max Ai)= Min(5, 6, 6) =5
Decision : Select A1

d) Laplace :

we associate equal probability for event say say ¼
Expected pay-offs are :

E(A1)= $\frac{3+5+8-1}{4} = \frac{15}{4} = 3.75$

E(A2)= $\frac{6+5+2+0}{4} = \frac{13}{4} = 3.25$

E(A3)= $\frac{0+5+6+4}{4} = \frac{15}{4} = 3.75$

A1 and A3 has the maximum expected pay-off
Decision : Select A1 or A3

e) Hurwicz :

D= α (Maximum in Ai) + (1 – α)(Minimum in Ai)

D(A1)=(0.4*8)+(0.6*-1) =2.6
D(A2)=(0.4*6)+(0.6*0) =2.4
D(A3)=(0.4*6)+(0.6*0) =2.4

D(A1) has the maximum
Decision : Select A1

Given : P(S1)=0.6 , P(S2)=0.1 , P(S3)=0.2 , P(S4)=0.1 in table (A) ,Find:
(f)Expected payoff criterion (E),
(g)Expected Opportunity Loss (EOL),
(k)Most likelihood criterion (ML)

Alternatives	States of nature			
	S1	S2	S3	S4
A1	3	5	8	-1
A2	6	5	2	0
A3	0	5	6	4
probability	0.6	0.1	0.2	0.1

➢ Decision making under Risk.

Expected payoff criterion (E)

E for A1 = (3*0.6)+(5*0.1)+(8*0.2)+(-1*0.1)= 3.8
E for A2 = (6*0.6)+(5*0.1)+(2*0.2)+(0*0.1)= 4.5
E for A3 = (0*0.6)+(5*0.1)+(6*0.2)+(4*0.1)= 2.1

E for A2 is greater
Decision : Select A2

f) Expected Opportunity Loss (EOL) ,

Loss table				
Alternatives	States of nature			
	S1	S2	S3	S4
A1	3	0	0	5
A2	0	0	6	4
A3	6	0	2	0
probability	0.6	0.1	0.2	0.1

ELO for A1 = (3*0.6)+(0*0.1)+(0*0.2)+(5*0.1)= 2.3
ELO for A2 = (0*0.6)+(0*0.1)+(6*0.2)+(4*0.1)= 1.6
ELO for A3 = (6*0.6)+(0*0.1)+(2*0.2)+(0*0.1)= 4

ELO for A2 is least
Decision : Select A2

g) Most likelihood criterion (ML)

High probability is P1=0.6 with the high payoff at A2=6

Decision : Select A2

(H.W)

Example: An agricultural company wants to decide which commodity should stock to get maximum profit. It was supplied with the following information. The probability that the monsoon will **be excess, normal and deficient is 0.40,0.30 and 0.30 .** The estimated profit or loss three commodities in respect of these different kinds of monsoon are:

Profit per 1 ton			
Monsoon	Excess	Normal	Deficient
Rice	10,000	-4,000	15,000
Wheat	4,000	-3,000	8,000
Maize	4,000	1,000	-1,000

Determine the optimal decision under each of the following decision criteria and show how you arrived at it:

(a) Maximax, (b) Maximin, (c) Minimax regret (savage criterion), (d) Equal likelihood (Laplace), (e) Hurwicz Alpha criterion α=0.8, (f)E, (g)EOL, (k)ML .

➢ Decision making under uncertainty.

1. Maximax criterion (OPTIMISM)

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
MAX	**15,000**	**8,000**	**4,000**

The maximum of column maxima is 15,000. Hence, the company should adopt Rice commodity.

2. Maximin criterion (pessimism).

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
Min	**-4,000**	**-3,000**	**-1,000**

The maximum of column minima is -1,000. Hence, the company should adopt Maize commodity.

3. Minimax regret.

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000

Note: From the given payoff matrix, develop an opportunity-loss (or regret) matrix as follows:
(i) Find the best payoff corresponding to each state of nature
(ii) Subtract all other payoff values in that row from this value.

Opportunity Loss Table

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000-10,000= 0	10,000-4,000= 6,000	10,000-4,000= 6,000
Normal	1,000-(-4,000)= 5,000	1,000-(-3,000) = 4,000	1,000-1,000= 0
Deficient	15,000-15,000= 0	15,000-8,000 = 7,000	15,000-(-1,000)= 16,000
MAX	**5,000**	**7,000**	**16,000**

Decision: Rice

4. Equal likelihood (Laplace).

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
Expected (average)	$\frac{1}{3}(10{,}000 - 4{,}000 + 15{,}000) = 7000$	$\frac{1}{3}(4{,}000 - 3{,}000 + 8{,}000) = 3000$	$\frac{1}{3}(4{,}000 - 1{,}000 + 1{,}000) = 1333.33$

Decision: Rice

5. Hurwicz Alpha criterion α=0.8.

Alternative / Monsoon	Rice	Wheat	Maize
Excess	10,000	4,000	4,000
Normal	-4,000	-3,000	1,000
Deficient	15,000	8,000	-1,000
weighted average: D_i	0.8(15,000)+0.2(-4,000)= 11,200	0.8(8,0000+0.2(-3,000)= 5,800	0.8*4,000+0.2(-1,000)= 3000

$D = \alpha\,(\textit{Maximum in column}) + (1-\alpha)(\textit{Minimum in column})$

Decision: Rice

- **Decision making under Risk.**

1. Expected payoff criterion (E) .

Alternative / Monsoon	Rice	Wheat	Maize	probability
Excess	10,000	4,000	4,000	0.4
Normal	-4,000	-3,000	1,000	0.30
Deficient	15,000	8,000	-1,000	0.30
E	0.4(10,000)+0.3(-4,000)+0.3(15,000)= 7300	0.4(4,000)+0.3(-3,000)+0.3(8,000)= 3100	0.4(4,000)+0.3(-1,000)+0.3(1,000)= 1600	

Decision: Rice

2. Expected Opportunity Loss (EOL).

Opportunity Loss Table

Alternative / Monsoon	Rice	Wheat	Maize	probability
Excess	0	6,000	6,000	0.4
Normal	5,000	4,000	0	0.30
Deficient	0	7,000	16,000	0.30
EOL	0.3(5,000) =1,500	0.4(6,000)+0.3(4,000)+0.3(7,000) =5,700	0.4(6,000)+0.3(16,000) =7,200	

Decision: Rice

3. Most likelihood criterion (ML)

Alternative / Monsoon	Rice	Wheat	Maize	probability
Excess	10,000	4,000	4,000	0.4
Normal	-4,000	-3,000	1,000	0.30
Deficient	15,000	8,000	-1,000	0.30

ML with high probability is P1=0.4 at A=rice with payoff =10,000

Decision: Rice