6. CONTINOUSE PROBABILITY DISTRIBUTIONS

Q1) Find the moment generating function for the general normal distribution.

Q2) Show that the moment generating function of the random variable X which is Chi square distribution with v degree of freedom is $M(t) = (1-2t)^{-v/2}$.

Q3) If X_1 and X_2 be independent r.v. that are chi-square dis. with v1 and v2 degrees of freedom, respectively.

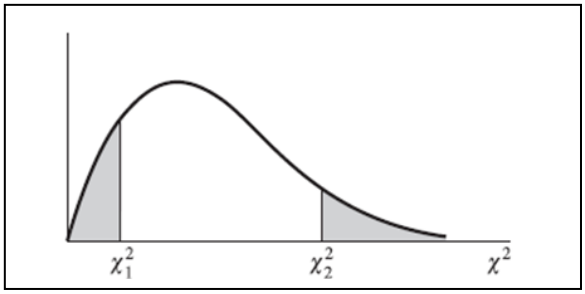
a) Show that the moment generating function of the random variable $Z = X_1 + X_2$ is $M(t) = (1-2t)^{-(v1+v2)/2}$
b) What you can say about the distribution of the random variable Z.

Q4) Show that the mean and variance of gamma distribution are given by

a) $\mu = \frac{\alpha}{\beta}$
b) $\sigma^2 = \frac{\alpha}{\beta^2}$

Q5) The grahp of chi-square distribution with 5 degrees of freedom is shown below. Find the values of χ_1^2, χ_2^2 for which

a. The shaded area on the right = 0.05,
b. The total shaded area = 0.05,
c. The shaded area on the left = 0.10,
d. The shaded area on the right = 0.01.



Q6) The graph of t- distribution with 9 degrees of freedom is shown below. Find the values of t_1, t_2 for which

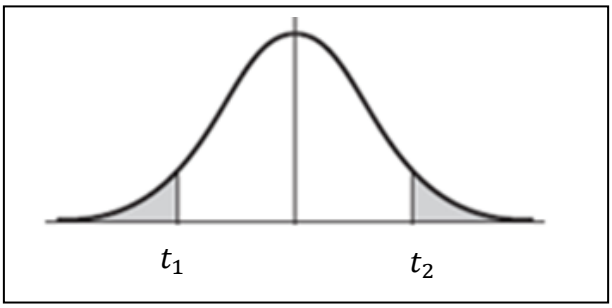
a. The shaded area on the right = 0.05,

b. The total shaded area = 0.05,

c. The total unshaded area = 0.99,

d. The shaded area on the left = 0.01,

e. The area on the left of t_2 = 0.90.



Q7) Let X be an exponential random variable with parameter $\theta = ln(3)$. Compute the following probability:$P(2 \leq X \leq 4)$.

Q8) Suppose the random variable has an exponential distribution with parameter $\theta = 1$. compute $P(X > 2)$.

Q9) What is the probability that a random variable X is less than its expected value, if X has an exponential distribution with parameter θ?

Q10) Identify the distribution of the r.v. from the moment generating function

(a) $M_x(t) = \frac{1}{1-2t}, t < 1/2$

(b) $M_x(t) = e^{3t+2t^2}$

(c) X, Y independent, $M_{X+Y}(t) = \left(\frac{2}{2-t}\right)^3, t < \frac{1}{2}, Y \sim Exp(2)$

Q11) X, Y independent, $M_{X+Y}(t) = \frac{e^{2t}-1}{2t-t^2}$, $X \sim Exp(2)$, what is the distribution of Y