

## Exercise

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Note: +S: slack variable, -S: surplus variable, *URS: unrestricted*,

**Q1: Put the following LP in the standard form:**

$$1- \text{Max } Z = X_1 - X_2 - 3X_3$$

Subject to

$$2X_1 + X_2 - X_3 \leq 2$$

$$X_1 - 3X_2 + 2X_3 \leq 3$$

$$X_1 + X_2 - X_3 \geq -2$$

$$X_1 \geq 0, X_2 \leq 0, X_3 \text{ URS}$$

The standard form

Replace  $X_2$  with  $(-X_2^-)$ , and Replace  $X_3$  with  $(X_3^+ - X_3^-)$

$$\text{Max } Z = X_1 + X_2^- - 3X_3^+ + 3X_3^-$$

Subject to:

$$2X_1 - X_2^- - X_3^+ + X_3^- + S_1 = 2$$

$$X_1 + 3X_2^- + 2X_3^+ - 2X_3^- + S_2 = 3$$

$$-X_1 + X_2^- + X_3^+ - X_3^- + S_3 = 2$$

$$X_1, X_2^-, X_3^+, X_3^-, S_1, S_2, S_3 \geq 0$$

$$2- \text{Min } Z = 80X_1 + X_2$$

Subject to

$$0.2X_1 + 0.32X_2 \leq 0.25$$

$$X_1 + X_2 = 1$$

$$X_1 \geq 0, X_2 \geq 0$$

$$\text{Min } Z = 80X_1 + X_2$$

Subject to

$$0.2X_1 + 0.32X_2 + S_1 = 0.25$$

$$X_1 + X_2 = 1$$

$$X_1, X_2, S_1 \geq 0$$

$$\mathbf{3- \text{Min } Z = 3X_1 + 8X_2 + 4X_3}$$

Subject to

$$X_1 + X_2 \geq 8$$

$$2X_1 - 3X_2 \leq 0$$

$$X_2 \geq 9$$

$$X_1 \geq 0, X_2 \geq 0, X_3 \text{ URS}$$

Replace  $X_3$  with  $(X_3^+ - X_3^-)$

The standard form

$$\text{Min } Z = 3X_1 + 8X_2 + 4(X_3^+ - X_3^-)$$

Subject to

$$X_1 + X_2 - S_1 = 8$$

$$2X_1 - 3X_2 + S_2 = 0$$

$$X_2 - S_3 = 9$$

$$X_1, X_2, X_3^+, X_3^-, S_1, S_2, S_3 \geq 0$$

**Q2: For the LP, answer the following questions?**

$$\mathbf{\text{Max } Z = 5X_1 + 4X_2}$$

Subject to

$$6X_1 + 4X_2 \leq 24$$

$$X_1 + 2X_2 \leq 6$$

$$X_1 \geq 0, X_2 \geq 0$$

- Express the problem in equation form.
- Determine the all basic solutions and classify them as feasible and infeasible.
- Use direct substitution in the objective function to determin the optimum basic feasible solution.
- Verify graphically that the solution obtained in (c) is the optimum LP solution.

The standard form

$$\text{Max } Z = 5X_1 + 4X_2$$

Subject to

$$6X_1 + 4X_2 + S_1 = 24$$

$$X_1 + 2X_2 + S_2 = 6$$

$$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0$$

Note:

(m) basic variables; (n-m) non-basic variables; (S) is slack variable.

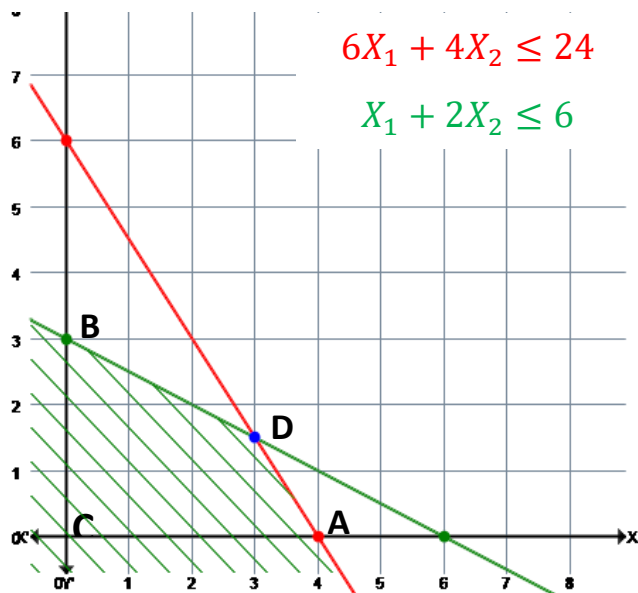
To find total number of basic solutions by use  $\binom{n}{m} = nC_m$  “combinations”

All basic solutions are not necessarily feasible.

We have **m=2** constraints and **n=4** variables, thus **n-m=2** Nonbasic variables (zero variables).

Total number of Basic solutions are  $\binom{4}{2} = 6$

Nonbasic Variables	Basic Variables	Basic Solution	Feasibility Status	Extreme point	Objective Value
$S_1, S_2$	$X_1, X_2$	3,1.5	Feasible	D	21
$S_2, X_2$	$X_1, S_1$	6,-12	Infeasible		_____
$S_1, X_2$	$X_1, S_2$	4,0	Feasible	A	20
$S_2, X_1$	$X_2, S_1$	3,12	Feasible	B	12
$S_1, X_1$	$X_2, S_2$	6,-6	Infeasible		_____
$X_1, X_2$	$S_1, S_2$	24,6	Feasible	C	0



$$\boxed{1} \quad S_1 = S_2 = 0$$

$$6X_1 + 4X_2 = 24$$

$$-6 * X_1 + 2X_2 = 6$$

$$(4 - 12)X_2 = 24 - 36$$

$$-8X_2 = -12$$

$$X_2 = 1.5$$

$$X_1 + 2(1.5) = 6$$

$$X_1 = 6 - 3 = 3$$

$$X_1 = 3$$

$$\boxed{2} \quad S_2 = X_2 = 0$$

$$X_1 + 2X_2 + S_2 = 6$$

$$X_1 = 6$$

$$6X_1 + 4X_2 + S_1 = 24$$

$$6(6) + 4(0) + S_1 = 24$$

$$S_1 = 24 - 36$$

$$S_1 = -12$$

$$\boxed{3} \quad S_1 = X_2 = 0$$

$$6X_1 + 4X_2 + S_1 = 24$$

$$6X_1 = 24$$

$$X_1 = 4$$

$$X_1 + 2X_2 + S_2 = 6$$

$$4 + 2(0) + S_2 = 6$$

$$S_2 = 2$$

$$\boxed{4} \quad S_2 = X_1 = 0$$

$$X_1 + 2X_2 + S_2 = 6$$

$$2X_2 = 6$$

$$X_2 = 3$$

$$6X_1 + 4X_2 + S_1 = 24$$

$$4(3) + S_1 = 24$$

$$S_1 = 24 - 12$$

$$S_1 = 12$$

$$\boxed{5} \quad S_1 = X_1 = 0$$

$$6X_1 + 4X_2 + S_1 = 24$$

$$4X_2 = 24$$

$$X_2 = 6$$

$$X_1 + 2X_2 + S_2 = 6$$

$$2(6) + S_2 = 6$$

$$S_2 = 6 - 12$$

$$S_2 = -6$$

$$\boxed{6} \quad X_1 = X_2 = 0$$

$$6X_1 + 4X_2 + S_1 = 24$$

$$S_1 = 24$$

$$X_1 + 2X_2 + S_2 = 6$$

$$S_2 = 6$$

**1- Max  $Z = 3X_1 + 2X_2$  H.W**

Subject to

$$2X_1 + 4X_2 \leq 8$$

$$X_1 + X_2 \leq 2$$

$$X_1 \geq 0, X_2 \geq 0$$

- a) Express the problem in equation form.
- b) Determine the all basic solutions and classify them as feasible and infeasible.
- c) Use direct substitution in the objective function to determin the optimum basic feasible solution.
- d) Verify graphically that the solution obtained in (c) is the optimum LP solution.