Exercise -3-

Using the graphical method, solve each of the following LPP

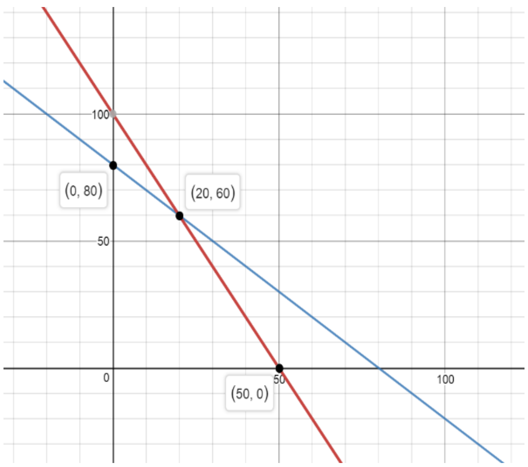
1- Max $Z = 50X_1 + 18X_2$ **(H.W)**

Subject to

$2X_1 + X_2 \leq 100$

$X_1 + X_2 \leq 80$

$X_1 \geq 0, X_2 \geq 0$



(X_1, X_2)	Z
(0,80)	1440
(20,60)	2080
(50,0)	2500

2- Max $Z = 10X_1 + 8X_2$

Subject to

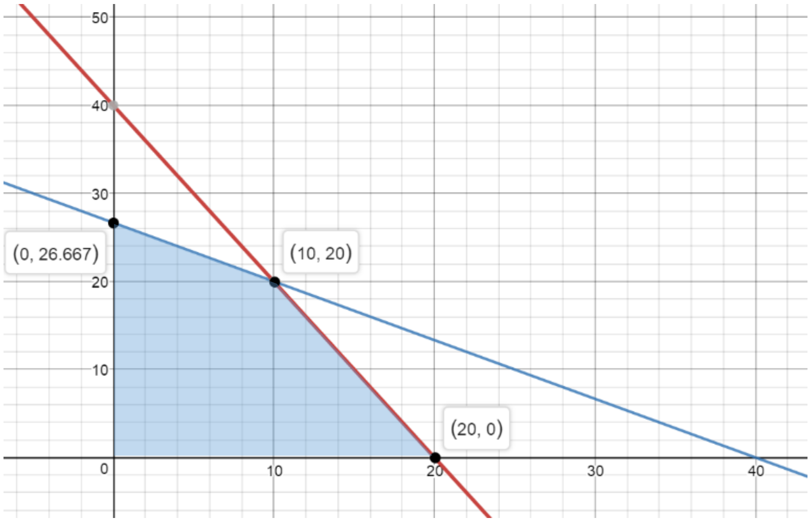
$2X_1 + X_2 \leq 40$

$2X_1 + 3X_2 \leq 80$

$X_1 \geq 0, X_2 \geq 0$

$2X_1 + X_2 = 40$ >> (0,40) and (20,0)

$2X_1 + 3X_2 = 80$ >> (0,26.667) and (40,0)



(X_1, X_2)	Z
(0,26.667)	213.33
(10,20)	260
(20,0)	200

3- Max $Z = 300X_1 + 400X_2$

Subject to

$5X_1 + 4X_2 \le 200$ $5X_1 + 4X_2 = 200$ >> (0,50) and (40,0)

$3X_1 + 5X_2 \le 150$ >> (0,30) and (50,0)

$5X_1 + 4X_2 \ge 100$ >> (0,25) and (20,0)

$8X_1 + 4X_2 \ge 80$ >> (0,20) and (10,0)

$X_1 \ge 0, X_2 \ge 0$

(X_1, X_2)	Z
(0,25)	10000
(0,30)	12000
(30.769,11.538)	13846.1
(40,0)	6000
(20,0)	12000

Q: Determine the optimality condition that will keep the optimum unchanged. Also, Determine the optimality rang for C1 and C2, assuming that the other coefficient is kept constant at its present value.?!

The optimal solution occurs at (30.769,11.538) intersection between (1) and (2) constraints.

$$(1)\, 5X_1 + 4X_2 \le 200$$
$$(2)\; 3X_1 + 5X_2 \le 150$$

$$\frac{-5}{4} \le \frac{-C_1}{C_2} \le \frac{-3}{5} \quad >> \quad -1.25 \le \frac{-C_1}{C_2} \le -0.6 \quad >> \quad 0.6 \le \frac{C_1}{C_2} \le 1.25.$$

suppose that coefficient C_2 is fixed at its current value of C_2=400, then the optimality range for C_1 is

$$\frac{3}{5} \le \frac{C_1}{400} \le \frac{5}{4}$$

$$240 \le C_1 \le 500$$

suppose that coefficient C_1 is fixed at its current value of C_1=300, then the optimality range for C_2 is

$$\frac{3}{5} \leq \frac{300}{C_2} \leq \frac{5}{4}$$

$$\frac{4}{5} \leq \frac{C_2}{300} \leq \frac{5}{3}$$

$$240 \leq C_2 \leq 500$$

the parameters (input data) of the model can change within certain units without causing the optimum solution to change (sensitivity analysis- change in the objective coefficients).

Q: If the objective function change to $Z = 350\, X_1 + 300\, X_2$, does the optimal solution will still unchanged?

To check $\frac{C_1}{C_2} = \frac{350}{300} = 1.167 \quad \in [0.6, 1.25]$

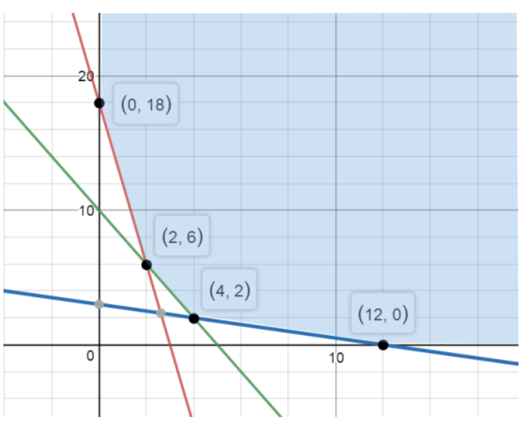
Thus, the optimal solution will still unchanged

4- Min $Z = 20X_1 + 40X_2$

Subject to

(1) $36X_1 + 6X_2 \geq 108$
(2) $3X_1 + 12X_2 \geq 36$
(3) $200X_1 + 100X_2 \geq 1000$

$X_1 \geq 0, X_2 \geq 0$



(X_1, X_2)	Z
(0,18)	720
(2,6)	280
(4,2)	160
(12,0)	240

Q: Determine the optimality condition that will keep the optimum unchanged. Also, Determine the optimality rang for C1 and C2, assuming that the other coefficient is kept constant at its present value.?!

The binding constraints are:

(2) $3X_1 + 12X_2 \leq 36$

(3) $200X_1 + 100X_2 \geq 1000$

Range of optimality

$\frac{-200}{100} \leq \frac{-C_1}{C_2} \leq \frac{-3}{12}$ >> $\mathbf{0.25 \leq \frac{C_1}{C_2} \leq 2}$

$0.25 \leq \frac{20}{C_2} \leq 2$ >> $4 \leq \frac{C_2}{20} \leq 0.5$ >> $\mathbf{80 \leq C_2 \leq 10}$ >> $\mathbf{10 \leq C_2 \leq 80}$

$0.25 \leq \frac{C_1}{40} \leq 2$ >> $\mathbf{10 \leq C_1 \leq 80}$

5- Min $Z = 120X_1 + 100X_2$ **(H.W)**

Subject to

$10X_1 + 5X_2 \leq 80$

$6X_1 + 6X_2 \leq 66$

$4X_1 + 8X_2 \geq 24$

$5X_1 + 6X_2 \leq 90$

$X_1 \geq 0, X_2 \geq 0$

Special cases in the Graphical Method:

1) Unbounded solution.

2) Infeasible/ No solution.

3) Multiple Optimal solution.

Q: Using the graphical method, solve each of the following LPP:

1-Unbounded Solution

1-Max $Z = 2X_1 + X_2$

Subject to:

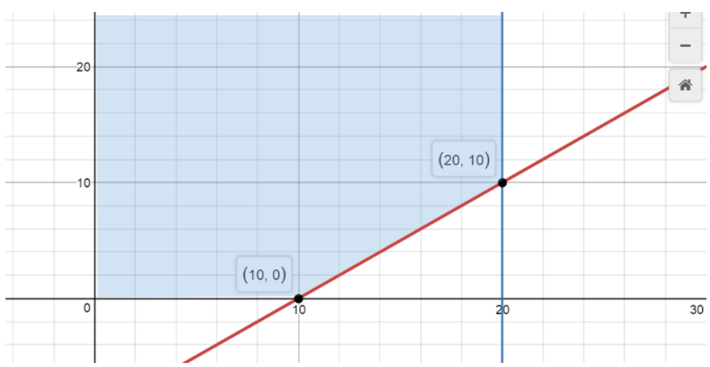
$X_1 - X_2 \leq 10$

$2X_1 \leq 40$

$X_1 \geq 0, X_2 \geq 0$

$X_1 - X_2 = 10$ >> (0,-10) and (10,0)

$2X_1 = 40$ >> $X_1 = 20$



The solution space is unbounded in direction of X_2, and the value of Z can be increased indefinitely.

HW 2-Max $Z = -2X_1 + 6X_2$

Subject to

$X_1 + X_2 \geq 2$

$X_2 - X_1 \leq 1$

$X_1 \geq 0, X_2 \geq 0$

2-Infeasible (No Solution)

1-Max $Z = 200X_1 + 300X_2$

Subject to

$0.2X_1 + 0.3X_2 \geq 120$

$0.1X_1 + 0.1X_2 \leq 40$

$0.5X_1 + 0.15X_2 \geq 90$

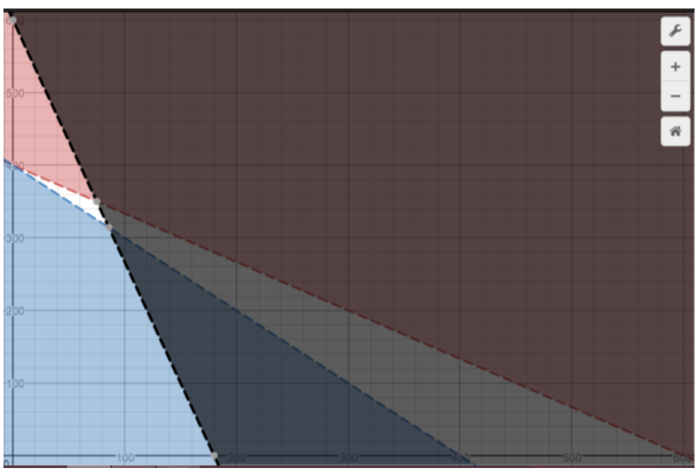
$X_1 \geq 0, X_2 \geq 0$

To determine tow point on each straight line

$0.2X_1 + 0.3X_2 = 120$ >> (0,400) and (600,0)

$0.1X_1 + 0.1X_2 = 40$ >> (0,400) and (400,0)

$0.5X_1 + 0.15X_2 = 90$ >> (0,600) and (180,0)



The problem is infeasible.

HW 2-Max $Z = 3X_1 + 2X_2$

Subject to

$2X_1 + X_2 \leq 2$

$3X_1 + 4X_2 \geq 12$

$X_1 \geq 0$, $X_2 \geq 0$

3-Multiple Optimal solution

6- Max $Z = 200X_1 + 400X_2$

Subject to

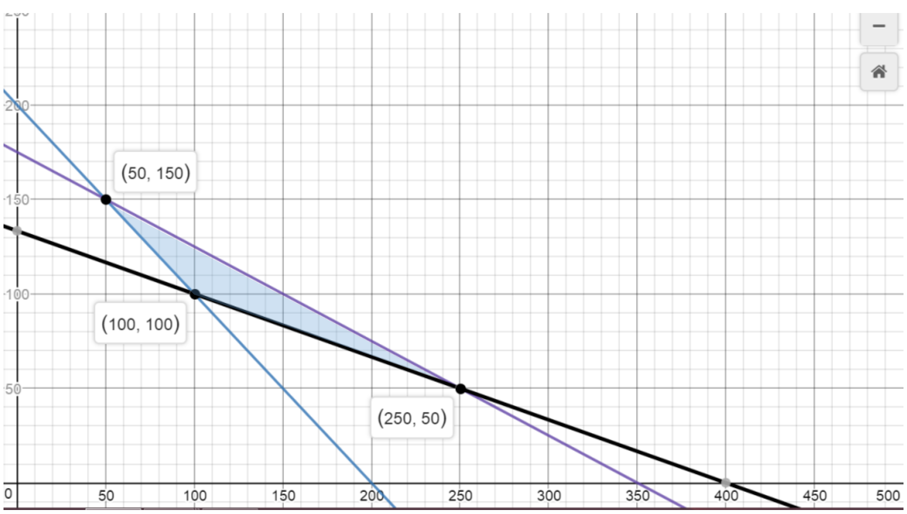
$(1) X_1 + X_2 \geq 200$
$(2) X_1 + 3X_2 \geq 400$
$(3) X_1 + 2X_2 \leq 350$
$(4) X_1 \geq 0, X_2 \geq 0$

To determine tow point on each straight line

$X_1 + X_2 = 200$ >> (0,200) and (200,0)

$X_1 + 3X_2 = 400$ >> (0,133.3) and (400,0)

$X_1 + 2X_2 = 350$ >> (0,175) and (350,0)



(-)* $X_1 + X_2 = 200$
$\underline{X_1 + 2X_2 = 350}$
$X_2 = 350 - 200 = 150$
$X_1 = 50$

(-)* $X_1 + X_2 = 200$
$\underline{X_1 + 3X_2 = 400}$
$2X_2 = 400 - 200$
$X_2 = 100, X_1 = 100$

$X_1 + 3X_2 = 400$
$\underline{(-) * \ X_1 + 2X_2 = 350}$
$X_2 = 400 - 350 = 50$
$X_1 = 250$

(X_1, X_2)	Z
(0,100)	60000
(50,150)	70000
(250,50)	70000

The problem has Multiple Optimal solution .

HW 2-Min $Z = 3X_1 + 2X_2$

Subject to

$-X_1 + X_2 \leq 2$

$3X_1 + 2X_2 \geq 12$

$X_1 \geq 0, X_2 \geq 0$