

Q1

The stopping distance (y , in feet) of a car was studied in relation to its velocity (x , in miles per hour (mph)). The table below lists the stopping distances at 6 different velocities.

Velocity (mph)	20.5	20.5	30.5	40.5	48.8	57.8
Stopping distance (ft)	15.4	13.3	33.9	73.1	113.0	142.6

- Plot y against x , and $z = \sqrt{y}$ against x .
- Compute the sample correlation coefficients of y with x , and z with x .
- Fit a linear regression model to y on x and examine the residuals (the differences between the \hat{y} values and the y values, i.e. $\hat{\varepsilon}_i = y_i - \hat{y}_i$). The estimate of σ in this case is $\hat{\sigma} = 7.563$.
- Fit a linear regression model to z on x and examine the residuals. The estimate of σ in this case is $\hat{\sigma} = 0.322$.
- Compute prediction intervals with a coverage probability of 0.95 for y and z when $x = 35$.
- Which model is better? Briefly explain why.

Q2

. A statistician suggests an estimator of β given by:

$$\tilde{\beta} = \frac{1}{n} \sum_{i=1}^n \frac{y_i}{x_i}$$

where n is the sample size and the model is $y_i = \beta x_i + \varepsilon_i$, $E(\varepsilon_i) = 0$ and $E(\varepsilon_i^2) = \sigma^2$. Show that $\tilde{\beta}$ is an unbiased estimator of β and that its variance is:

$$\text{Var}(\tilde{\beta}) = \frac{\sigma^2}{n^2} \left(\sum_{i=1}^n \frac{1}{x_i^2} \right)$$

under certain assumptions. What extra assumptions have you used?

Q3

The records on the wholesale food price index, x , and the retail food price index, y , over $n = 12$ years yield the following data:

$$\bar{x} = 92.1667, \quad \bar{y} = 93.1667, \quad \sum_{i=1}^{12} (x_i - \bar{x})(y_i - \bar{y}) = 200.6667,$$

$$\sum_{i=1}^{12} (x_i - \bar{x})^2 = 255.6667 \quad \text{and} \quad \sum_{i=1}^{12} (y_i - \bar{y})^2 = 179.6667.$$

(a) Estimate the simple linear regression model:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i.$$

with the usual assumptions on ε_i .

(b) Estimate $\sigma^2 = \text{Var}(\varepsilon_i)$.

(c) Find a 95% prediction interval for the retail price index when the wholesale food price index is 84.