# EXERCISES <br> STAT - 109 <br> BIOSTATISTICS 

Q1: For each of the following variables indicate whether it is quantitative or qualitative variable:
(a)The blood type of some patient in the hospital.
(b) Blood pressure level of a patient.
(c) Weights of babies born in a hospital during a year.
(d) Gender of babies born in a hospital during a year.
(e) The distance between the hospital to the house .
(f) Under-arm temperature of day-old infants born in a hospital.

Q2: For each of the following situations, answer questions (a) through (d):
(a) What is the population?
(b) What is the sample in the study?
(c) What is the variable of interest?
(d) What is the type of the variable?

Situation A: A study of 300 households in a small southern town revealed that if she has school-age child present.

Situation B: A study of 250 patients admitted to a hospital during the past year revealed that, Distance the patient live away from the hospital .

## Q3:Choose the right answer:

## 1-The variable is a

a. subset of the population.
b. parameter of the population.
c. relative frequency.
d. characteristic of the population to be measured.
e. class interval.

## 2-Which of the following is an example of discrete variable

a. the number of students taking statistics in this term at KSU.
b. the time to exercise daily.
c. whether or not someone has a disease.
d. height of certain buildings.
e. Level of education.

3-Which of the following is not an example of discrete variable
a. the number of students at the class of statistics.
b. the number of times a child cry in a certain street.
c. the time to run a certain distance.
d. the number of buildings in a certain street.
e. number of educated persons in a family.

4-Which of the following is an example of qualitative variable
a. the blood pressure.
b. the number of times a child brush his/her teeth.
c. whether or not someone fail in an exam.
d. Weight of babies at birth.
e. the time to run a certain distance.

## 5-The continuous variable is a

a. variable with a specific number of values.
b. variable which can't be measured.
c. variable takes on values within intervals.
d. variable with no mode.
e. qualitative variable.

## 6- which of the following is an example of continuous variable

a. The number of visitors of the clinic yesterday.
b. The time to finish the exam.
c. The number of patients suffering from certain disease.
d. Whether or not the answer is true.

## 7- The discrete variable is

a-qualitative variable.
b-variable takes on values within interval.
c-variable with a specific number of values.
d-variable with no mode.
8-Which of the following is an example of nominal variable :
a-age of visitors of a clinic.
b-The time to finish the exam.
c-Whether or not a person is infected by influenza.
d-Weight for a sample of girls .

## 9-The nominal variable is a

a-A variable with a specific number of values
b-Qualitative variable that can't be ordered.
c-variable takes on values within interval.
d-Quantitative variable .
10-Which of the following is an example of nominal variable :
a-The number of persons who are injured in accident.
b-The time to finish the exam.
c-Whether or not the medicine is effective.
d-Socio-economic level.

## 11-The ordinal variable is :

a-variable with a specific number of values.
b-variable takes on values within interval.
c-Qualitative variable that can be ordered.
d -Variable that has more than mode.

## Exercise \# 2

Q1.A study was conducted in which they measured incidental intracranial aneurysms (IIAs) in 159 patients.The researchers examined complications and concluded that IIAs can be safely treated without causing mortality and with a lower complications rate than previously reported.

The following table represent the sizes ( in millimeters) of the 159 IIAs in the sample:

| IIAs size | frequency | Cumulative <br> Frequency | Relative <br> frequency | Cumulative <br> Relative <br> Frequency | Percentage <br> frequency |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0-4$ | 29 |  | 0.182 |  |  |  |
| $5-9$ | 87 |  | ----- |  |  |  |
| $10-14$ | ----- |  | 0.163 |  |  |  |
| $15-19$ | 10 | ------ | 0.0629 |  |  |  |
| $20-24$ | 4 |  | 0.025 |  | ------ |  |
| $25-29$ | 1 |  | 0.006 |  |  |  |
| $30-34$ | 2 |  | 0.013 | ----- |  |  |
| Total | ----- |  |  |  |  |  |

Complete the table , then answer the following questions

1. The variable is $\qquad$ , the type of variable is
2. The number of patient with IIAs size between $10-14$ is $\qquad$
3. The number of patient with IIAs size less than or equal 19 is $\qquad$
4. The relative frequency patient with IIAs size between 5-9 is $\qquad$
5. The proportion of patient with IIAs size less than 15 is $\qquad$
6. The percentage of patient with IIAs size between $15 \mathbf{- 2 9}$ is $\qquad$
7. The true class interval of (20-24) is $\qquad$
8. Width is
9. Maximum value is $\qquad$

Q2: The following table shows the number of hours 45 hospitals patients slept following the administration of a certain anesthetic .

| True Class interval | Frequency | Midpoint |
| :---: | :---: | :---: |
| $0.5-5.5$ | 21 |  |
| $5.5-10.5$ | 16 | ----- |
| --------- |  |  |
| $15.5-20.5$ | 2 |  |
| Total | 45 |  |

Answer the following questions:

1. The variable is $\qquad$
The type of variable is $\qquad$
2. The sample size is $\qquad$
3. The midpoint for the interval $5.5 \mathbf{- 1 0 . 5}$ is $\qquad$
4. The number of patients spend less than or equal 15.5 hour is
$\qquad$
5. The relative frequency of patients spend between $0.5-10.5$ hour is
$\qquad$
6. The class interval for the true class interval ( $5.5-10.5$ ) is
(where $d=0.5$ )
7. The percentage of patients spend more than 10.5 hour is $\qquad$
8. Width is $\qquad$

## H.W

In a study of physical endurance of male college freshman , The following table show the composite endurance scores based on 155 exercise routines were collected

| endurance <br> scores | frequency | Relative <br> frequency |
| :---: | :---: | :---: |
| $115-134$ | 6 | 0.039 |
| $135-154$ | 7 | 0.045 |
| $155-174$ | ---- | 0.103 |
| $175-194$ | 31 | 0.200 |
| $195-214$ | 37 | ---- |
| $215-234$ | ---- | 0.181 |
| $235-254$ | 18 | 0.116 |
| $255-275$ | 8 | 0.052 |
| $275-294$ | 3 | 0.019 |
| $295-314$ | 1 | 0.006 |
| Total | ---- | 1 |

## Answer the following questions :

1. The variable is $\qquad$ The type of variable is $\qquad$
2. The population is $\qquad$
3. The midpoint for the interval 195-214 is $\qquad$
4. The number of males with endurance score more than or equal $\mathbf{2 3 5}$ is
$\qquad$
5. The proportion of males with endurance score between 155-234 is
$\qquad$
6. The true class interval for class interval (215-234) is $\qquad$
7. The percentage of males with endurance score between (275-294) is
$\qquad$
8. Width is $\qquad$
9. Minimum value is $\qquad$

Q3: For a sample of patients, we obtain the following graph for approximate hours spend without pain after certain surgery .


Answer the following questions:

1. The type of the graph is $\qquad$
2. The variable is $\qquad$
The type of the variable is $\qquad$
3. The sample size is $\qquad$
4. The number of patients spend a round 2 hours without pain is
$\qquad$
5. The percent of patients spend 3.5 hours or more without pain is
$\qquad$
6. The number of patients stayed the longest time without pain is
7. The lowest number of hours spent without pain is $\qquad$
8. Width is $\qquad$
H.W : For a sample of Saudi women , we obtain the following graph for the serum cholesterol ( in $\mathrm{mmol} / \mathrm{I}$ ) .


Answer the following questions:

1. The type of the graph is $\qquad$
2. The variable is $\qquad$ The type of the variable is $\qquad$
3. The sample size is $\qquad$
4. The number of Saudi women with more than 6 serum cholesterol is $\qquad$
5. The percent of Saudi women between 4 and 6 serum cholesterol is
$\qquad$
6. The serum cholesterol with the lowest percentage is between the interval $\qquad$
7. Width is $\qquad$

## Exercise \# 3

## How to use calculator

Mode (3:stat ) ___ Then ___ (1: 1-VAR)
يظهر جدول لإدخال البيانات
Example: 2, 4,6, 9
$2=, 4=, 6=, 9=$
نضنط (AC)
To find mean, standard deviation and variance
نضغط
Shift ( 1)
وتختار الرقم
5:Var ( or 4:Var)
ويظهر في الشاشة

| 1:n | $2: \bar{X}$ |
| :---: | :---: |
| 3: xon (or $\sigma_{x}$ ) | 4: $x \sigma_{n-1}\left(\right.$ or $\left.S_{x}\right)$ |

## For mean: Push 2

## For Sample standard deviation :Push 4

For Population standard deviation :Push 3
*To find sample variance = square (Sample standard deviation)

* To find Population variance = square (Population standard deviation)

For each of the data sets in the following exercises compute:
a) The mean
b) The median
c) The mode
d) The range
e) The variance
f) The standard deviation
g) The coefficient of variation

1- Porcellini et al. studied 13 HIV- positive patients who were treated with highly active antiretroviral therapy (HAART) for at least 6 months. The CD4 T cell counts at baseline a sample of 13 subjects are listed below:

| 230 | 205 | 313 | 207 | 227 | 245 | 173 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 58 | 103 | 181 | 105 | 301 | 169 |  |

2- For the following population

$$
\begin{array}{llllllllllllll}
41 & 37 & 51 & 32 & 48 & 53 & 44 & 32 & 38 & 42 & 46 & 48 & 49 & 51
\end{array}
$$

3- Shair and Jasper investigated whether decreasing the venous return in young rats would affect ultrasonic vocalizations (USVs). Their research showed no significant change in the number of ultrasonic vocalizations when blood was removed from either the superior vena cava or the carotid artery. Another important variable measured was the heart rate (bmp) during the withdrawal of blood. The data below presents the heart rate of a sample of seven rat pups from the experiment involving the carotid artery:

$$
\begin{array}{lllllll}
500 & 570 & 560 & 570 & 450 & 560 & 570
\end{array}
$$

4- Cardosi et al. performed a 4-year retrospective review of 102 women undergoing radical hysterectomy for cervical or endometrial cancer. Gatherer-associated urinary tract infection was observed in a sample of 12 of the subjects. Below are the numbers of postoperative days until diagnosis of the infection for each subject experiencing an infection:

$$
\begin{array}{llllllllllll}
16 & 10 & 49 & 15 & 6 & 15 & 8 & 19 & 11 & 22 & 13 & 17
\end{array}
$$

## 5- H.W

According to strach et al, hamstring tendon grafts have been the "weak link" in anterior cruciate ligament reconstruction. In a controlled laboratory study, they compared two techniques for reconstruction: either an interference screw or a central sleeve and screw on the tibial side. For eight cadaveric knees, the measurements below represent the required force (in Newtons) at which initial failure of graft strands occurred for the central sleeve and screw technique:

$$
\begin{array}{llllllll}
172.5 & 216.63 & 212.62 & 98.97 & 66.95 & 239.76 & 19.57 & 195.72
\end{array}
$$

a) The mean $=152.84$
b) The median $=184.11$
c) There is no mode
d) Range $=220.19$
e) Variance $=6494.724$
f) The Standard Deviation= 80.5899
g) $\mathrm{C} . \mathrm{V}=52.73 \%$

## 6- H.W

The purpose of a study by Nozawa et al. was to evaluate the outcome of surgical repair of pars interarticularis defect by segmental wire fixation in young adults with lum-bar spondylolysis. The authors found that segmental wire fixation historically has been successful in the treatment of nonathletes with spondylolysis, but no information existed on the results of this type of surgery in athletes. In a retrospective study, the authors found 20 subjects who had the surgery between 1993 and 2000. For these subjects, the data below represent the duration in months of follow-up care after the operation:

| 103 | 68 | 62 | 60 | 60 | 54 | 49 | 44 | 42 | 41 |
| ---: | ---: | ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 38 | 36 | 34 | 30 | 19 | 19 | 19 | 19 | 17 | 16 |

a) The mean $=41.5$
b) The median $=39.5$
c) The mode $=19$
d) Range $=87$
e) Variance $=490.264$
f) The Standard Deviation= 22.1419
g) $\mathrm{C} . \mathrm{V}=53.35 \%$

## 7- H.W

Butz et al. evaluated the duration of benefit derived from the use of noninvasive positivepressure ventilation by patients with amyotrophic lateral sclerosis on symptoms, quality of life, and survival. One of the variables of interest is partial pressure of arterial carbon dioxide ( PaCO 2). The values below ( mm of Hg ) reflect the result of baseline testing on 30 subjects as established by arterial blood gas analyses:

| 40.0 | 47.0 | 34.0 | 42.0 | 54.0 | 48.0 | 53.6 | 56.9 | 58.0 | 45.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 54.5 | 54.0 | 43.0 | 44.3 | 53.9 | 41.8 | 33.0 | 43.1 | 52.4 | 37.9 |
| 34.5 | 40.1 | 33.0 | 59.9 | 62.6 | 54.1 | 45.7 | 40.6 | 56.6 | 59.0 |

a) The mean $=47.416$
b) The median $=46.35$
c) The mode is 33.54
d) Range $=29.6$
e) Variance $=76.537$
f) The Standard Deviation $=8.7485$
g) $\mathrm{C} . \mathrm{V}=18.45 \%$

## Exercise \# 3

## How to use calculator to find measures

(mean , standard deviation and variance)
Example: Find sample mean, SD, variance :

$$
2,4,6,9
$$

Step 1: Turn on Statistics mode :
Press MODE


Press number 3 to choose " STAT "


Step 2: Enter data:
Press 1 to choose "1-VAR"


For the first value, press the number followed by =


Enter the rest of the data


Press AC to return to the main screen


Step 3: Calculate Mean and S.D :

Press SHIFT followed by 1


Press 4 to select "Var "


Press 2 to calculate mean
Press 4 to calculate sample standard deviation

To find variance: square the value of $S D$ press the square button $x^{2}$, then Equal $=$
the results of our example :
mean:

sample SD:

variance:


## Exercise \#4

*** In a study of violent victimization of women a d men, Porcerilli et al. (A-2) collected information from 679 women and 345 men aged 18 to 64 years at several family practice centers in the metropolitan Detroit area. Patients filled out a health history questionnaire that included a question about victimization. The following table shows the sample subjects categories are defined as no victimization, partner victimization (and not by others), victimization by person other than partners (family member, friends or strangers) and those who reported multiple victimization

|  | No <br> Victimization | Partners | Non-partners | Multiple <br> partners | Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Women | 611 | 34 | 16 | 18 | 679 |
| Men | 308 | 10 | 17 | 10 | 345 |
| Total | 919 | 44 | 33 | 28 | 1024 |

Suppose we pick a subject at random from this group. Find

1. The probability that this subject will be a woman is $\qquad$
2. The probability that the subject will be a woman and have experienced partner abuse is $\qquad$
3. Suppose we picked a man at random knowing that he is a man, then the probability that he experienced abuse from non- partner. $\qquad$
4. The probability that is a man or someone who experienced abuse from partner. $\qquad$
5. The relation between being a man and being a woman is
a. Disjoint
b. exhaustive and disjoint
c. independent
d. exhaustive
****Fernando et al. (A-s) studied drug-sharing among injection drug users $u=i n$ South Bronx in New York City. Drug user in New York City use the term "split a bag" or "get down a bag" to refer the practice of dividing a bag of heroin or other injectable substances. A common practice includes splitting drugs after they are dissolved in a common cooker, a procedure with considerable HIV risk. Although this practice is common, little is known about the prevalence of such practice, the researchers asked injection drug user in four neighborhoods in the South Bronx if they ever "got down on" drug in bags or shots. The results classified by gender and splitting practice are given below:

| Gender | Split Drugs | Never Split Drugs | Total |
| :---: | :---: | :---: | :---: |
| Male | 349 | 324 | 673 |
| Female | 220 | 128 | 348 |
| Total | 569 | 452 | 1021 |

## If a person picked at random. Find the probability that

1. He is never split drugs and is female. $\qquad$
2. She he admits to splitting drug, given that she is female $\qquad$
3. He is not a man is $\qquad$
4. $\left(\right.$ Male $^{\mathrm{C}} \cap$ Split Drugs $)=$
5. $\mathrm{P}($ Males U Split Drugs $)=$
6. $P($ Male $\mid$ Split Drugs $)=$
7. $\quad \mathbf{P}($ Male $)=$
8. The relation between being a man and Never split drug is
a. Disjoint
b. exhaustive
c. independent
d.exhaustive

## **** suppose that dental clinic has $\mathbf{1 2}$ nurses classified as follows

## The experiments is to randomly choose one of these nurses. Consider the following events:

$\mathrm{C}=$ the chosen nurse has children.
$\mathrm{N}=$ the chosen nurse works night shift.

| Nurse | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Has <br> Children | Yes | No | No | No | No | Yes | No | No | Yes | No | No | No |
| Works <br> at nigh | No | No | Yes | Yes | Yes | Yes | No | No | Yes | Yes | Yes | Yes |

If we pick a nurse at random,then
1- The probabilities that the chosen nurse has children is .....

2- The probabilities that the chosen nurse works night shift is......

3- The probabilities that the chosen nurse has children and works night shifts

4- The probabilities that the chosen nurse has children and doesn't work night shifts $\qquad$

5- Are having children and work at night disjoint?

6- Are having children and work at night exhaustive?

7- Are having children and work at night independent?

## H.W

****Laveist and Nuru-Jeter (A-4) conducted a study determined if doctor-patient race concordance was associated with greater with care. Toward that end, they collected a national sample of African=American, Caucasian, Hispanic and Asian-American respondents. The following table classifies the race of the subject as well as the race of their physician:

| Patient's Race |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Caucasian | African American | Hispanic | AsianAmerican | Total |
|  | White | 779 | 436 | 406 | 475 | 1796 |
|  | African American | 14 | 162 | 15 | 5 | 196 |
|  | Hispanic | 19 | 17 | 128 | 2 | 166 |
|  | AsianAmerican Islander | 68 | 75 | 71 | 203 | 417 |
|  | Other | 30 | 55 | 56 | 4 | 145 |
|  | Total | 910 | 745 | 676 | 389 | 2720 |

## If we a randomly selected subject, then

1. The probability that a randomly selected subject will have an

Asian/Pacific-Islander physicians. $\qquad$
2. The probability that an African-America subject will have an AfricanAmerican Physician is $\qquad$
3. The probability that a randomly selected subject will have as AsianAmerican and have an Asian/Pacific-Islander Physician. $\qquad$
4. The probability that a subject chosen at random will be Hispanic or have a Hispanic Physician $\qquad$
5. The relation between Physician Race is Hispanic and Patient's Race is African American is
a.Disjoint
b. exhaustive
c. independent
d.exhaustive

## Exercise \#5

Q1:A medical research team wishes to assess the usefulness of a certain symptom (call is $S$ ) in the diagnosis of a particular disease. In a random sample of $\mathbf{7 5 5}$ patients with the disease. 744 reported having the symptom. In an independent random sample of 1380 subjects without the disease, 21 reported having that they had the symptom.

|  | With Disease <br> $(\mathrm{D})$ | Without Disease <br> $\left(\mathrm{D}^{\mathrm{C}}\right)$ | Total |
| :---: | :---: | :---: | :---: |
| Positive (T) | 744 | 21 | 765 |
| Negative $\left(\mathrm{T}^{\mathrm{C}}\right)$ | 31 | 1359 | 1390 |
| Total | 775 | 1380 | 2155 |

## 1. what is false positive?

(A)Probability that result of the test is positive given that patient has disease.
(B)Probability that result of the test is negative given that patient has disease.
(C)Probability that result of the test is positive given that patient doesn't have disease.
(D)Probability that result of the test is negative given that patient doesn't have disease.
2. What is false negative?
(A)Probability that result of the test is positive given that patient has disease.
(B)Probability that result of the test is negative given that patient has disease.
(C)Probability that result of the test is positive given that patient doesn't have disease.
(D)Probability that result of the test is negative given that patient doesn't have disease.
3. Compute the sensitivity of the symptom?
4. Compute the specificity of the symptom?
5. Suppose it is know that the rate of the disease in the general population is . 001 . What is the predictive value negative of the symptom?
6. What is the predictive value negative of the symptom?
$\underline{\mathbf{H} . \mathbf{W}}$ :Find the predictive value positive and the predictive value negative for the symptom for the following hypothetical disease $.0001, .01$, and .1 ?

Q2: In article entitled 'Bucket-Handle Meniscal Tears of the Knee: Sensitivity and Specificity of MRI signs" Dorsay and Helms (A-6) performed a retrospective study of $\mathbf{7 2}$ knees scanned by MRI. One of the indicators they examined was the absence of the "bow tie sign" in the MRI as evidence of a bucket-handle or "bucket-handle type" tar of the meniscal. In the study, surgery confirmed that 43 of the $\mathbf{7 3}$ cases were bucket-handle tears. The cases may be cross-classified by "bow tie sign" status and surgical results as follows:

|  | Tears Surgically <br> Confirmed <br> (D) | Tears Surgically <br> confirmed as not <br> Present ( $\dot{\mathrm{D}}^{\mathrm{C}}$ ) | Total |
| :---: | :---: | :---: | :---: |
| Positive Test (absent <br> bow tie sign (T) | 38 | 10 | 48 |
| Negative Test ( bow <br> tie sign Present) <br> ( $\dot{\mathrm{T}}^{\mathrm{C}}$ ) | 5 | 15 | 23 |
| Total | 43 | 28 | 71 |

## 1. what is false positive?

(A)Probability that result of the test is positive given that patient has disease.
(B)Probability that result of the test is negative given that patient has disease.
(C)Probability that result of the test is positive given that patient doesn't have disease.
(D)Probability that result of the test is negative given that patient doesn't have disease.

## 2. What is false negative?

(A)Probability that result of the test is positive given that patient has disease.
(B)Probability that result of the test is negative given that patient has disease.
(C)Probability that result of the test is positive given that patient doesn't have disease.
(D)Probability that result of the test is negative given that patient doesn't have disease.
3. Compute the sensitivity of the symptom?
4. Compute the specificity of the symptom?
5. Suppose it is know that the rate of the disease in the general population is $\mathbf{0 0 1}$. What is the predictive value negative of the symptom?
6. What is the predictive value negative of the symptom?
*******************************************************

## H.W

Repeat exercise question 2 at disease rate $0.01 \mathrm{P}(\mathrm{D})=0.01$

## Chapter (4) :

## Q1: For the following probability distribution

| $\mathbf{x}$ | $\mathbf{0}$ | $\mathbf{1}$ | 2 | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| . $\mathbf{f}(\mathbf{x})$ | $\mathbf{0 . 0 5}$ | $\mathbf{0 . 1 5}$ | $\mathbf{k}$ | $\mathbf{0 . 2 5}$ | $\mathbf{0 . 3}$ | $\mathbf{0 . 1}$ |

1. The value of $K$ is $\qquad$
2. The value of $x$ with the highest probability is $\qquad$
3. $\mathbf{P}(\mathbf{X}<3)=$
4. $P(1 \leq X<4)=$
5. Mean of $x$ is
6. Standard deviation of X is
7. Variance is
***************************************************
Q2: For a population of families, Let
$X=$ the number of children in primary school.
We randomly choose one and the cumulative distributed is given below
8. $P(X=2)=$
9. $P(X=4)=$
10. $\mathbf{P}(1.5 \leq X \leq 2)=$
11. $P(X>2)=$

| $\mathbf{X}$ | $\mathbf{P}(\mathbf{X} \leq \mathbf{x})$ |
| :---: | :---: |
| $\mathbf{0}$ | $\mathbf{0 . 1 2}$ |
| $\mathbf{1}$ | $\mathbf{0 . 3 6}$ |
| $\mathbf{2}$ | $\mathbf{0 . 7 2}$ |
| $\mathbf{3}$ | $\mathbf{0 . 9 5}$ |
| $\mathbf{5}$ | $\mathbf{1}$ |

5. Mean is
6. Variance is

## Q3:

Given the following probability distribution of a discrete random variable $\mathbf{X}$ representing the number of defective teeth of the patient visiting a certain dental clinic

1. The value of the $k$ is $\qquad$
2. $\mathbf{P}(\mathrm{x}<3)=$
3. $P(X \leq 3)=$
4. $P(X<6)=$
5. $P(X=3.5)=$
6. Probability that the patient has at least $\mathbf{4}$ defective Teeth $\qquad$

| $\mathbf{X}$ | $\mathbf{P}(\mathbf{X}=\mathbf{x})$ |
| :---: | :---: |
| $\mathbf{1}$ | $\mathbf{0 . 2 5}$ |
| 2 | $\mathbf{0 . 3 5}$ |
| 3 | $\mathbf{0 . 2 0}$ |
| $\mathbf{4}$ | $\mathbf{0 . 1 5}$ |
| $\mathbf{5}$ | $\mathbf{K}$ |

7. Probability that the patient has at most 2 defective Teeth $\qquad$
8. The expected number of defective teeth $($ Mean $)=$ $\qquad$
9. The variance of $X$ is $\qquad$

How to use the calculator to find the mean and variance for the discrete random variable

1/ Shift $\rightarrow 9 \rightarrow 1 \rightarrow=\rightarrow \mathrm{AC}$
2/ Shift $\rightarrow$ mode $\rightarrow$ click to the down


## To find the mean

4/ Shift $\rightarrow 1 \rightarrow$ 4:Var or 5:Var $\rightarrow$ 2: $\bar{X} \longrightarrow=$
To find the Standard deviation (SD )
5/ Shift $\rightarrow 1 \rightarrow$ 4:Var or 5:Var $\rightarrow$ 3: $\sigma_{X} \rightarrow=$
To find the Variance
$\sigma_{X}{ }_{X}=(\mathbf{S D})^{2}$

## Exercise 6

## Question 1:

The same survey data base cited shows that $32 \%$ of U.S. adults indicated that they have tested for HIV at some point in their life. Consider a simple random sample of 15 adults selected at that time. Find the probability that the number of adults who have been tested for HIV in the sample would be:
(a) Three.
(b) Less than five.
(c) Between five and nine. Inclusive.
(d) More than five, but less than 10 .
(e) Six or more.
(f) Mean equals
(g) Variance equals

## Question 2:

Coughlin et al. estimated the percentage of woman living in border counties along the southern United States with Mexico (designated counties in California, Arizona, New Mexico, and Texas) who have less than a high school education to be 19. Suppose we select three women at random. Then
(a) The probability that the number of women with less than a highschool education is zero $\qquad$
(b) The probability that the number of women with less than a highschool education is one $\qquad$
(c) The probability that the number of women with less than a highschool education is two or fewer. $\qquad$
(d) The probability that the number of women with less than a highschool education is two or three $\qquad$
(e) Mean equal $\qquad$
(f) Standard deviation equal $\qquad$

## Home work:

In a survey of nursing students pursuing a master's degree, 75 percent stated that they expect to be promoted to a higher position within one month after receiving the degree. If this percentage holds for the entire population, find, for a sample of 15 , the probability that the number expecting a promotion within a month after receiving their degree is:
(a) Six.
(b) At last seven.
(c) No more than five.
(d) Between six and nine, inclusive.

## Question 3:

Singh et al. (A-7) Looked at the occurrence of retinal capillary hemangiona patients with von hippel - Lindau (VHL) disease. RCH is a benign vascular tumor of the retina. Using a retrospective consecutive case series review, the researchers found that the number of RCH tumor incidents followed a Poisson distribution with $\underline{\lambda=4 \text { tumors per eye for patients with VHL. Using }}$ this model.

The probability that in a randomly patient with VHL
(a) There will be exactly five occurrences of tumors per eye equals...
(b) There are more than five occurrences of tumors per 2-eyes equals...
(c) There are fewer than five occurrences of tumors per 10-eyes equals....
(d) There are between five and seven occurrences tumors per eye, inclusive........
(e) The mean(per eye)
(f) The variance (Per 2-eyes )

## Question 4:

In a certain population an average of 13 new cases of esophageal cancer are diagnosed each year. If the annual incidence of esophageal cancer follows a Poisson distribution, find the probability that the number of newly diagnosed cases of esophageal cancer will be.
(a) Exactly 10 in year $\qquad$
(b) At last eight in a month $\qquad$
(c) Between nine and 15 , inclusive in a week $\qquad$
(d) Fewer than seven in 2 years $\qquad$
(e) Mean( in week)
(f) Standard deviation(in year)

## Home work:

If the mean number of serious accident per month in a large factory (where the number of employs remains constant) is 2 ,
(a) The probability that in the current year there will be exactly seven accidents is $\qquad$
(b) The probability that in the current week there will be ten or more accident is $\qquad$
(c) The probability that in the current day there will be no accidents is
(d) Mean(In a week) $\qquad$
(e) Variance(In a 3 years )

Standard Normal Table
Areas Under the Standard Normal Curve


| 2 | -0.09 | -0.08 | -0.07 | -0.00 | -0.05 | -0.04 | -0.03 | -0.02 | -0.01 | -0.00 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3.50 | 0.00017 | 0.00017 | 0.00018 | 0.00019 | 0.00019 | 0.00020 | 0.00021 | 0.00022 | 0.00022 | 0.00023 | 50 |
| -3 | 0.0002 | 0.00025 | 0.0002 | 0.0002 | 0.000 | 0.0 | 0.000 | 0.0 | 0.00032 | 0.00034 | -3.40 |
| -3 | 0.00035 | 0.00036 | 0.00038 | 0.00039 | 0.000 | 0.000 | 0.000 | 0.00045 | 0.00047 | 0.00048 | 30 |
|  | 0.00050 | 0.00052 | 0.00054 | 0.0005 | 0.0005 | 0.000 | 0.00062 | 0.000 | 0.00066 | 0.00069 | -3.20 |
| -3.10 | 0.00 | 0.00074 | 0.0 | 0.0 | 0.0 | 0.0008 | 0.0 | 0.00090 | 0.00094 | 0.00 | 10 |
| -3.00 | 0.0 | 0.00104 | 0.0 | 0.00111 | 0.00114 | 0.00118 | O. | 0.00126 | 0.00131 | 0.00135 | -3.00 |
|  | 0.00 | 0.00144 | 0.0 | 0.0 | 0.0 | 0.00164 | 0.0 | 0.00175 | 0.00181 | 0.00187 | -2.90 |
| -2. | 0.00193 | 0.00 | 0.002 | 0.00 | 0.002 | 0.00 | 0.00 | 0.00240 | 0.00248 | 0.00256 | 80 |
| -2 | 0.00264 | 0.00272 | 0.0028 | 0.00289 | 0.00298 | 0.00307 | 0.0031 | 0.0032 | 0.00336 | 0.00347 | -2.70 |
| -2.60 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.00440 |  |  | -2.60 |
| -2.50 | 0.0 | 0.0 | 0.0 |  | 0.0 | 0.0 | 0.0 | . 0 | 0.00604 | . | - |
| -2 | 0.00 | 0.0 | 0.00 | 0.0 | 0.0 | 0.0 | 0.007 | 0.00776 |  | 0.00820 | -2.40 |
| -2 | 0.00 | 0.0 | 0.00 | 0.0 | 0.0 | 0.0 | 0.009 | 0.0 | 0.01044 | 0.01072 | 30 |
| -2 | 0.01 | 0.0 | 0.0 | 0.0 | 0.0122 | 0.012 | 0.01287 | 0.0132 | 0.01355 | 0.01390 | 20 |
| -2.10 | 0.0 | 0.0 | 0.0 |  | 0.0 | 0.0 | 0.0 | 0.01700 | 3 |  | -2.10 |
| -2.00 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. | 0.02169 | 0.02222 | 0.02275 | -2.00 |
| -1 | 0.02 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.02743 | 0.02807 | 0.0 | -1.90 |
| -1 | 0.02 | 0.0 | 0.03 |  | 0.032 | 0.03 | 0.0 | 0.0 | 0.03 | 0.03593 | 80 |
| -1 | 0.03673 | 0.03 | 0.0383 | 0.03920 | 0.04 | 0.04093 | 0.04182 | 0.0427 | 0.04363 | 0.04457 | -1.70 |
| -1 | 0.0 | 0.0464 | 0.0474 | 0.04846 | 0.0494 | 0.050 | 0.0515 | 0.0526 | 0.05370 | 0.05480 | 左 |
| -1.50 | 0.0 | 0.0 | 0.0 | 0. | 0.0 | 0.0 | 0.0 | 0.06426 | 0.06552 | 0.06681 | -1.50 |
| -1.4 | 0.06 |  | 0.0 |  | 0.0 | 0.0 | 0.0 | 0.07780 | 0.07927 | 0.0 | -1.40 |
| -1 |  |  |  |  |  |  | 0.0 |  | 0.09510 | 0.09680 | 30 |
| -1 | 0.09 | 0.1 | 0.1 | 0.1 | 0.10565 | 0.107 | 0.10935 | 0.1 | 0.11314 | 0.11507 | -1.20 |
| -1.10 | 0.1 | 0.11900 | 0.1 | 0.12302 | 0.1250 | 0.1 | 0.1 | 0.13136 | 0.13350 | 0.135 | - |
| -1.00 | 0.1 | 0.1 | 0.1 |  | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.15 | 00 |
| -0.90 | 0.1 | 0.1 | 0.1 |  | 0.1 | 0.1 | 0.1 | 0.17879 | 0.1 | 0.1 | 90 |
| -0.80 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 0.2 | 7 | 0.21186 | 80 |
| -0, | 0.2 | 0.2 | 0.2206 | 0.2 | 0.2 | 0.2 | 0.2 | 0.23576 | 0.2 | 0.2 | 70 |
| -0.60 | 0.2 | 0.2 | 0.2 | 0.25463 | 0.2578 | 0.261 | 0.2 | 0.267 | 0.27093 | 0.27425 | 60 |
| -0.50 | 0.27 | 0.28 | 0.28 | 0.28 | 0.291 | 0.29 | 0.2 | 0.30153 | 0.30503 | 0.308 | . 50 |
| -0.40 | 0.3120 | 0.31561 | 0.3191 | 0.3 | 0.32630 | 0.329 | 0.3336 | 0.337 | 0.3409 | 0.34458 | 40 |
| -0.30 | 0.34827 | 0.35197 | 0.355 | 0.35942 | 0.3631 | 0.36 | 0.37070 | 0.37448 | 0.37828 | 0.382 | -0.30 |
| -0.20 | 0.3859 | 0.38974 | 0.393 | 0.39743 | 0.4012 | 0.405 | 0.4090 | 0.41294 | 0.41683 | 0.42074 | 20 |
| -0.10 | 0.42465 | 0.42858 | 0.43251 | 0.43644 | 0.44038 | 0.44433 | 0.44828 | 0.45224 | 0.45620 | 0.46017 | $-0.10$ |
| -0.00 | 0.46414 | 0.46812 | 0.472 | 0.47608 | 0.48 | 0.4 | 0.48803 | 0.49202 | 0.49601 | 0.50000 | -0.0 |

## Standard Normal Table (continued) <br> Areas Under the Standard Normal Curve

|  | 0.00 | 0.01 | 0.02 |  |  | 0.05 |  | 0.07 |  | 0.09 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 0.50000 | 0.50399 | 0.50798 | 0.51197 | 0.51595 | 0.51994 | 0.52392 | 0.52790 | 0.53188 | 86 | 0.00 |
|  | 0.53983 | 0.54380 | 0.54776 | 0.551 | 0.5556 | 0.55962 | 0.56356 | 0.56749 | 0.57142 | 0.57535 | 0.10 |
|  | 0.57926 | 0.58317 | 0.58706 | 0.59095 | 0.5948 | 0.5987 | 0.60257 | 0.60642 | 0.61026 | 0.61409 | 0.20 |
|  | 0.6179 | 0.6217 | 0.625 | 0.629 | 0.633 | 0.636 | 0.6 | 0.64 | 0.648 | 0.65173 | 0.30 |
|  | 0.65542 | 0.65910 | 0.6627 | 0.666 | 0.67003 | 0.673 | 0.6 | 0.68 | 0.684 | 0.68793 | 0.40 |
|  | 0.69146 | 0.69497 | 0.6984 | 0.7019 | 0.7054 | 0.708 | 0.71226 | 0.715 | 0.71904 | 0.72240 | 0.50 |
|  | 0.72575 | 0.72907 | 0.7 | 0.73565 | 0.73891 | 0.7 | 0.7453 | 0.7485 | 0.7517 | 0.75490 | 0.60 |
|  | 0.75804 | 0.7611 | 0.7 | 0.7 | 0.7 |  | 0.7 | 0.7 | 0.78230 | 24 | 70 |
|  | 0.78814 | 0.79103 | 0.7938 | 0.7 | 0.7995 | 0.8 | 0.805 | 0.807 | 0.81057 | 0.813 | 0.80 |
|  | 0.8159 | 0.81859 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8314 | 0.83 | 0.836 | 0.83891 | 0.90 |
|  | 0.84 | 0.8437 | 0.8 | 0.8 | 0.8 | 0.8 | 0.85543 | 0.8576 | 0.85993 | 0.86214 | 1.00 |
|  | 0.8 | 0.8 | 0.8 | 0.87076 | 0.8 | 0.87493 | 0.87698 | 0.87900 | 0.8 | 88298 | 1.10 |
|  | 0.88 | 0.88 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.89 | 0.90 | 20 |
|  | 0.90320 | 0.9049 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.916 | 0.91774 | 1.30 |
|  | 0.91924 | 0.9207 | 0.9 | 0.9 | 0.925 | 0.9 | 0.92785 | 0.9292 | 0.93056 | 0.93189 | 40 |
|  | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.94408 | 1.50 |
|  | 0.94 | 0.94 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.953 | 0.95449 | 1.60 |
|  | 0.95 | 0.95 | 0.9 | 0.9 | 0.9 | 0.9 | 0.96080 | 0.9 | 0.962 | 0.96327 | 1.70 |
|  | 0.96 | 0.964 | 0.9 | 0.9 | 0.96 | 0.9 | 0.9 | 0.969 | 0.969 | 0.97062 | 1.80 |
|  | 0.97 | 0.97193 | 0.9 | 0.9 | 0.9 | 0.9 | 0.97500 | 0.97 | 0.9761 | 0.97670 | 1.90 |
|  | 0.97 | 0.97 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.98 | 0.98169 |  |
|  | 0.9 | 0.9 | 0.9 |  | 0.9 |  | 0.98461 |  | 0.98537 | 4 |  |
|  | 0.98610 | 0.98 | 0.9 | 0.9 | 0.9 | 0.9 | 0.98809 |  | 0.988 | 0.98899 |  |
|  | 0.98 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.99111 | 0.99134 | 0.99158 |  |
|  | 0.9 | 0.99 | 0.9 | 0.99245 | 0.9 | 0.9 | 0.99305 | 0.99324 | 0.99343 | 1 |  |
|  | 0.9 | 0.9 | 0.9 |  |  |  | 0.99477 | 0.9 | 0.99506 | 20 |  |
|  | 0.99 | 0.9 | 0.9 | 0.9 |  |  |  | 0.99621 | 0.99632 | 3 | 2.60 |
| 2.70 | 0.99 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.99720 | 0.99728 | 736 | 2.70 |
| 2.80 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.99788 | 0.99795 | 0.99801 | 99807 | 2.80 |
| 2.90 | 0.99 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.99846 | 0.99851 | 0.99856 | 0.99861 | 2.90 |
| 3.00 | 0.99 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.99896 | 0.9990 | 3.00 |
| 3.10 | 0.9990 | 0.9990 | 0.99910 | 0.9 | 0.9 | 0.999 | 0.9992 | 0.999 | 0.99926 | 0.99929 | 3.10 |
| 3.20 | 0.99931 | 0.99934 | 0.99936 | 0.999 | 0.9 | 0.999 | 0.99944 | 0.99946 | 0.99948 | 0.99950 | 3.20 |
| 3.30 | 0.99 | 0.9995 | 0.9 | 0.9 | 0.9 | 0.9996 | 0.9996 | 0.999 | 0.99964 | 0.99965 | 3.30 |
| 3. | 0.9996 | 0.99968 | 0.99 | 0.9 | 0.9 | 0.99972 | 0.99973 | 0.99974 | 0.99975 | 0.99976 | 3.40 |
|  | 0.99977 | 0.99978 | 0.9997 | 0.999 | 0.99980 | 0.9998 | 0.99981 | 0.99982 | 0.99983 | 0.99983 |  |

## Exercise \#7

## Question(1):

Given the standard normal distribution find :
a) The area under the curve between $Z=0$ and $Z=1.43$
b) The probability that a $Z$ picked at random will a value between $Z=$ 2.87 and $Z=2.64$
c) Area to the left of $Z=1.43$ equals
d) Area to the right of $\mathrm{Z}=2$ equals
e) $P(Z>0.55)=$
f) $P(Z \geq-0.55)=$
g) $P(Z<-2.33)=$
h) $\mathbf{P}(\mathrm{Z}<-5)=$
i) $P(Z \geq 6.5)=$
j) $\mathbf{P}(\mathbf{Z} \leq k)=0.0054$, then the value of $k=\ldots .$.
k) $P(Z \geq k)=0.03836$, then the value of $k=\ldots .$.

1) $P(-2.67<Z \leq k)=0.97179$, then the value of $k=$ $\qquad$

## Home work:

a) $\mathbf{P}(\mathrm{Z}<2.33)$
b) $P(-1.96<Z \leq 1.96)$
c) $\mathbf{P}(-2.58<\mathrm{Z}<2.58)$
d) $P(-1.65<Z \leq 1.65)$
e) $\mathbf{P}(\mathrm{Z}=0.74)$

## Question (2):

For another subject (a 29 - year - old male) in the study Disking et al. (A-10), acetone lev- els were normally distributed with a mean of 870 and a standard deviation of 211 ppb. Then
a) The probability that on a given day the subject's acetone between 600 and 1000 ppb is. $\qquad$
b) The probability that on a given day the subject's acetone over 900 ppb is $\qquad$
c) The probability that on a given day the subject's acetone under 500 ppb is $\qquad$
d) Percentage over 900 ppb is $\qquad$
e) If we take population of $\mathbf{1 0 , 0 0 0}$ how many would expect be over 900?

## Question (3):

In the study of fingerprints an important quantitative characteristic is the total ridge count for the 10 fingers of an individual Suppose that the total ridge counts of individuals in a certain population are approximately normally distributed with a mean of 140 and a standard deviation of 50 . Then
a) The probability that an individual picked at random from this population will have a ridge count of 200 or more is .....
b) The probability that an individual picked at random from this population will have a ridge count of Less than 100 is. $\qquad$
c) The probability that an individual picked at random from this population will have a ridge count of between 100 and 200 is .....
d) The percentage that individual picked at random from this population will have a ridge count of Less than 100 is.
f) In a population of $\mathbf{1 0 0 , 0 0 0}$ people how many would you expect to have a ridge count of $\mathbf{2 0 0}$ or more?

## Home work:

One of the variables collected in North Carolina Birth Registry data is pounds gained during pregnancy. According to data from the entire registry for 2001. The number of pounds gained during pregnancy was approximately normally distributed with a mean of 30.23 pounds and a standard deviation of 13.84 pounds. Calculate the probability that a randomly selected mother in North Carolina in 2001 gained
a) Less than 15 pounds during pregnancy is
b) More than 40 pounds is $\qquad$
c) Between 14 and 40 pounds is $\qquad$
d) Percentage that mother in North Carolina in 2001 gained between 14 and 40 pounds is $\qquad$
e) If we take population of 10,000 how many would expect mother in North Carolina in 2001 gained between 14 and 40 pounds is $\qquad$

## Exercise \#8

## Question 1:

The National Health and Nutrition Examination Survey of 1988-1994(NHANES III, A-1) estimated the mean serum cholesterol level for U.S. females aged 20-74 years to be $204 \mathrm{mg} / \mathrm{dl}$. The estimate of the standard deviation was approximately 44. Using these estimates as the mean $\mu$ and the standard deviation $\sigma$ for the U.S. population, consider the sampling distribution of the sample mean based on samples of size 50 drawn from women of this age group. Find:

1) The mean of the sampling distribution $\bar{X}$
2) The standard error of $\bar{X}$
3) $\mathbf{P}(\bar{X}<200)$
4) $\mathbf{P}(\mathbf{1 7 0}<\bar{X}<195)$
5) $\mathbf{P}(\bar{X}<207)$
6) $\mathbf{P}(\bar{X}>190)$

## Question 2:

If the uric acid values in normal adult males are approximately normally distributed with a mean and standard deviation of 5.7 and 1 mg percent, respectively, for a sample of size 9 find:

1) The mean of $\bar{X}$
2) The standard error of $\bar{X}$
3) The probability that the mean of the uric acid values is greater than 6.
4) The probability that the mean of the uric acid values is between 5 and 6 .
5) The probability that the mean of the uric acid values is less than 5.8

## Question 3(H.W):

Given a normally distributed population with mean of 100 and a standard deviation of 20, find the following based on a sample of size 16 :

1) The mean of $\bar{X}$
( answer =100)
2) The variance of $\bar{X}$ $($ Answer $=25)$
3) The standard error of $\bar{X}$
$($ Answer $=5)$
4) $\mathrm{P}(\bar{x} \geq 100)=$
$($ Answer $=0.5)$
5) $\mathrm{P}(\bar{x} \leq 110)$
$($ Answer $=0.97725)$
6) $\mathrm{P}(96<\bar{x}<108)$
$($ Answer $=0.73334)$

## H.W

Given $\mu=50, \sigma=16$, and $n=64$, find:

1) $\mathrm{P}(45<\bar{x}<55) \quad$ (Answer $=0.98899)$
2) $\mathrm{P}(\bar{x}>53) \quad($ Answer $=0.06681)$
3) $\mathrm{P}(\bar{x}<47) \quad($ Answer $=0.06681)$
4) $\mathrm{P}(49<\bar{x}<56) \quad($ Answer $=0.69011)$

## Question 4:

In a study, the data about the serum cholesterol level in U.S. females are given in the following table:

| Population | Age | Mean | Standard <br> Deviation |
| :---: | :---: | :---: | :---: |
| A | $30-59$ | 189 | 34.7 |
| B | $20-29$ | 183 | 37.2 |

Suppose we select a simple random sample of size 50 independently from each population, then:

1) The mean of $\bar{x}_{A}-\bar{x}_{B}$ is:
2) The standard error of $\bar{x}_{A}-\bar{x}_{B}$ is:
3) The distribution of $\bar{x}_{A}-\bar{x}_{B}$ is:
4) The probability that the difference between sample means $\bar{x}_{A}-\bar{x}_{B}$ will be more than 8 .

## Question 5:

In a study, the calcium levels in men and women ages 60 years or older are summarized in the following table:

|  | Mean | Standard <br> Deviation |
| :---: | :---: | :---: |
| Men | 797 | 482 |
| Women | 660 | 414 |

If we take a random sample of 40 men and 35 women, then:

1) The mean of $\bar{x}_{1}-\bar{x}_{2}$ is:
2) The variance of $\bar{x}_{1}-\bar{x}_{2}$ is:
3) The distribution of $\bar{x}_{1}-\bar{x}_{2}$ is:
4) The probability of obtaining a difference between sample means $\bar{x}_{1}-\bar{x}_{2}$ of 100 mg or less

## Question 6:

Smith et al. (A-5) performed a retrospective analysis of data on 782 eligible patients admitted with myocardial infarction to a 46-bed cardiac service facility. Of these patients, 248 reported a past myocardial infarction. Suppose 50 subjects are chosen at random from the population, what is:

1) The mean of $\hat{P}$
2) The variance of $\hat{P}$
3) The standard error of $\hat{P}$
4) The distribution of $\hat{P}$
5) The probability that over 40 percent would report previous myocardial infarctions?

## Question 7:

Researchers estimated that 64 percent of U.S. adults ages 20-74 were overweight or obese. Use this estimate as the population proportion for U.S. adults ages 20-74. If 125 subjects are selected at random from the population, what is:

1) The mean of $\hat{P}$
2) The variance of $\hat{P}$
3) The standard error of $\hat{P}$
4) The distribution of $\hat{P}$
5) The probability that 70 percent or more would be found to be overweight or obese?

## H.W

given a population in which $\mathrm{p}=0.6$ and a random sample from this population of size 100 find:

1) $\mathrm{P}(\hat{P} \geq 0.65)$
$($ Answer $=0.15386)$
2) $\mathrm{P}(0.56<=\leq \hat{P} \leq 0.63) \quad$ (Answer $=0.52296)$
3) $\mathrm{P}(\hat{P} \leq .58)$
$($ Answer $=0.3409)$

## H.W

It is known that 35 percent of the members of a certain population suffer from one or more chronic diseases. What is the probability that in a sample of 200 subjects drawn at random from this population 40 or more will have at least one chronic disease?
$($ Answer $=0.03754)$

## Question 8:

In a study, the Census Bureau stated that for Americans in the age group 18 to 24 years, 64.8 percent had private health insurance. In the age group 2534years, the percentage was 72.1 . Assume that these percentages are the population parameters in those age groups for the United States. Suppose we select a random sample of 250 Americans from the 18-24 age group and an independent random sample of 200 Americans from the age group 25-34; find:

1) The mean of $\hat{P}_{1}-\hat{P}_{2}$
2) The standard error of $\hat{P}_{1}-\hat{P}_{2}$
3) The distribution of $\hat{P}_{1}-\hat{P}_{2}$
4) $\mathrm{P}\left(\widehat{P}_{1}-\hat{P}_{2} \leq 0.3\right)$
5) The probability that $\hat{P}_{1}-\hat{P}_{2}$ is less than 6 percent.

## H.W

From the results of a survey conducted by the U.S. Bureau of Labor Statistics (A-9), it was estimated that 21 percent of workers employed in the Northeast participated in health care benefits programs that included vision care. The percentage in the South was 13 percent. Assume these percentages are population parameters for the respective U.S. regions. Suppose we select a simple random sample of size 120 northeastern workers and an independent simple random sample of 130 southern workers. Find:

1) The mean of $\hat{P}_{1}-\hat{P}_{2}$ (Answer $=0.08$ )
2) The standard error of $\hat{P}_{1}-\hat{P}_{2} \quad$ (Answer $=0.0475$ )
3) The distribution of $\hat{P}_{1}-\hat{P}_{2} \quad$ (Answer $=\mathrm{N}(0.08,0.0023)$ )
4) What is the probability that the difference between sample proportions, $\hat{P}_{1}-\hat{P}_{2}$, will be between 0.04 and 0.20 .
$($ Answer $=0.79385)$

## Exercises \#9

## Question 1:

We wish to estimate the average number of heart beats per minute for a certain population. The average number of heartbeats per minute for a sample of 49 subjects was found to be 90 . Assume that these 49 patients constitute arandom sample, and that the population is normally distributed with a standard deviation of 10 .
(a) The point estimate of the mean $(\mu)$ is
(b) The standard error of sample mean $\bar{X}$ is
(c) The reability coefficient is ( at 95\% C.I)
(D) Construct the $95 \%$ confidence interval for the mean $(\mu)$ of the heart beats per minute?

## Question 2:

The concern of a study by beynnon et al. (A-4) were nine subjects with chronic anterior cruciate ligament (ACL) tears. One of the variables of interest was the laxity of the anteroposterior, where higher values indicate more knee instability. The researchers found that among subjects with ACLdeficient knees, the mean laxity value was 17.4 mm with a standard deviation of 4.3 mm . Assuming normally distributed:
(a) The point estimate of the population mean is
(b) The reability coefficient is (at $90 \%$ C.I)
(C) Construct the 90 percent confidence interval for the mean of the population from which the 9 subjects may be presumed to be s random sample?

The upper limit is
(d) What is the precision of the estimate (margin of error ) ${ }^{\text {! }}$ precision of the estimate $=$
(e) What assumptions are necessary for the validity of the confidence interval you constructed?

## H.W. 1:

We wish to estimate the mean serum indirect bilirubin level of 4-day-old infants. The mean for a sample of 16 infants was found to be $5.98 \mathrm{mg} / 100 \mathrm{cc}$. assume that bilirubin levels in 4 -day-old infants are approximately normally distributed with a standard deviation of $3.5 \mathrm{mg} / 100 \mathrm{cc}$.
(a) What is the point estimate of the mean $(\mu)$ ? $\quad($ Answer $=5.98)$
(b) The reability coefficient is $\left(\right.$ at $98 \%$ C.I) $\quad\left(\right.$ Answer $\left.=\boldsymbol{Z}_{\mathbf{0 . 9 9}}=\mathbf{2 . 3 2 5}\right)$
(C) Construct the $98 \%$ confidence interval for the mean $(\mu)$ of the serum indirect bilirubin level?
(1) The value of $\alpha$ is $\ldots \mathbf{0 . 0 2} \ldots$.
(2) Lower limit is

$$
\text { (Answer : } \bar{X}-Z_{0.975} \frac{\sigma}{\sqrt{n}}=5.98-2.325 \frac{3.5}{\sqrt{16}}=5.98-\mathbf{2 . 0 3 4 4}=\mathbf{3 . 9 4 5 6} \text { ) }
$$

## H.W. 2:

A sample of 16 ten-year-old girls had a mean weight of 71.5 and a standard deviation of 12 pounds, respectively. Assuming normally:
(a) What is the point estimate of the mean? $\quad($ Answer $=71.5)$
(b) Construct the 99 percent confidence interval for the mean of the population?
(Answer : $\bar{X} \pm \boldsymbol{t}_{0.995} \frac{s}{\sqrt{n}}=71.5 \pm 2.947 \frac{12}{\sqrt{16}}=71.5 \pm \mathbf{8 . 8 4 1}=(\mathbf{6 2 . 6 5 9}, \mathbf{8 0} .341)$
(c) What is the precision of the estimate? $\quad($ Answer $=\mathbf{8 . 8 4 1})$
(d) What assumptions are necessary for the validity of the confidence interval you constructed?
(Answer: normal , n small , $\sigma$ unknown )

## Question 3:

Chan et al. (A-9) developed a questkonnaire to assess knowledge of prostate cancer. There was a total of 36 questions to which respondents could answer "agree", "disagree", or "don't know". Scores could range from 0 to 36 . The number of Caucasian study participants was 185 , and the number of African-American was 86 . The mean scores for Caucasian study participants was 20.6 , while the mean scores for African-American men was 17.4. The population standard deviation for Caucasian study participants and African-American men equal of 5.8.
(a) What is the point estimate of $\left(\mu_{\text {Caucasian }}-\mu_{\text {African-American }}\right)^{\text {! }}$
(b) Construct the 99 percent confidence interval for the difference between the population mean scores for Caucasian study participants and the population mean scores for African-American men?
(1) The value of $\alpha$ is
(2) The reability coefficient is
(3) The precision of the estimate is $\qquad$
(4) The upper limit is $\qquad$
(c) What assumptions are necessary for the validity of the confidence interval you constructed?

## Question 4:

Transverse diameter measurements on the hearts of adult males and females gave the following results:

| Group | Sample size | $\overline{\boldsymbol{x}}(\mathbf{c m})$ | $\boldsymbol{s}(\mathbf{c m})$ |
| :---: | :---: | :---: | :---: |
| Males | 12 | 13.21 | 1.05 |
| Females | 9 | 11.00 | 1.01 |

Assume normally distributed populations with equal variances.
(a) What is the point estimate of $\left(\mu_{\text {Males }}-\mu_{\text {Females }}\right)^{\text {! }}$ ?
(b) What is the value of $S_{p}^{2!}$
(c) the value of degree of freedom is ...
(d) Construct the 90 percent confidence interval for the difference between the population mean of diameter the hearts for males and the population mean of diameter the hearts for females?
(1) The reability coefficient is
(2) What is the precision of the estimate?
(3) The lower limit is

## H.W. 3:

Twenty-four experimental animals with vitamin $D$ deficiency were divided equally into two groups. Group 1 received treatment consisting of a diet provided vitamin D. the second group was not treated. At the end of the experimental period, serum calcium determinations were made with the following results:

|  | $\overline{\boldsymbol{x}}(\boldsymbol{m g} / \mathbf{1 0 0} \boldsymbol{m l})$ | $\boldsymbol{s}(\boldsymbol{m g} / \mathbf{1 0 0} \mathbf{m l})$ |
| :---: | :---: | :---: |
| Treated group | 11.1 | 1.5 |
| Untreated group | 7.8 | 2.0 |

Assume normally distributed populations with equal variances.
(a) What is the point estimate of $\left(\mu_{\text {Treated }}-\mu_{\text {Untreated }}\right)^{\boldsymbol{\rho}}$
$($ Answer $=3.3)$
(b) What is the precision of the point estimate( at $90 \%$ C.I) ${ }^{\boldsymbol{@}}$
$($ Answer $=1.239)$
(c) What is the value of $S_{p}^{2}$ ?
$($ Answer $=3.125)$
(d) Construct the 90 percent confidence interval for the difference between the population mean of vitamin $D$ deficiency for males and the population mean of vitamin $D$ deficiency for females?
$($ Answer $=(2.061,4.539)$

## Question 5:

In a study by von zur Muhlenet al. (A-16), 136 subjects with syncope or near syncope were studied. Syncope is the temporary loss of consciousness due to a sudden decline in blood flowto the brain. Of these subjects, 75 alsoreported having cardiovascular disease.
(a) What is the point estimate of $p$ ?
(b) What is the estimated standard error of the sample proportions?
(c) Construct a 99 percent confidence interval for the population proportion of subjects with syncope or near syncope who also have cardiovascular disease?

## H.W. 4:

In a simple random sample of 125 unemployed male high-school dropouts between the ages of 16 and 21 , inclusive, 88 stated that they were regular consumers of alcoholic beverages.
(a) What is the point estimate of $p$ ? $\quad($ Answer $=0.704)$
(b)Construct a 95 percent confidence interval for the population proportion?
$($ Answer $=(0.624,0.6598)$

## Question 6:

Horwitz et al. (A-18) studied on persons who were identified by court records from 1967 to 1971 as having experienced abuse or neglect. For a control group, they located 510 subjects who as children attended the same elementary school and lived within a five-block radius oftbose in the abused/neglected group. In the abused/neglected group, and control group, 114 and 57 subjects, respectively, had developed antisocial personality disorders over their lifetimes.
(a) What is the value of $\hat{p}_{\text {abused } / \text { neglected }}$ ?
(b) What is the value of $\hat{p}_{\text {control }}$ ?
(c) What the point estimate of $p_{\text {abused } / \text { neglected }}-p_{\text {control }}$ ?
(d) What is the estimated standard error of the difference between sample proportions
$\hat{p}_{\text {abused } / \text { neglected }}-\hat{p}_{\text {control }}$ ?
(e) Construct a 95 percent confidence interval (C.I.) for the difference between the preparations of subjects developed antisocial personaliy disorders one might expect to find in the populations of subjects from which the subjects of this study may be presumed to have been drawn ( $\left.p_{\text {abused } / \text { neglected }}-p_{\text {control }}\right)$ ?

## H.W. 5:

In a study on patients with cancer, we have the following data:

| Group | $\mathbf{n}$ | Smoking | Not smoking |
| :---: | :---: | :---: | :---: |
| Men | 50 | 41 | 9 |
| Women | 40 | 22 | 18 |

(a) The point estimate of the difference between smoking proportions of men and women $p_{\text {Men }}-p_{\text {Women }} ? \quad($ Answer $=0.27)$
(b) What is the estimated standard error of the difference between sample proportions
$\hat{p}_{\text {Men }}-\hat{p}_{\text {Women }} ?($ Answer $=0.0956)$
(c) What is $99 \%$ C.I. for $p_{\text {Men }}-p_{\text {Women }}$ ?
1)The reability coefficient is

$$
\text { ( Answer= } \left.Z_{0.995}=2.575\right)
$$

2)What is the precision of the estimate?
(Answer= 0.24617 )
3) The upper limit is
((Answer= 0.51617)

## Exercise \#10

Q1:A study was made of a random sample of 25 records of patients seen at a chronic disease hospital on an outpatient basis, the mean number of outpatient visits per patient was 4.8 with standard deviation was 2 . Can it be concluded from these data that the population mean is greater than four visits per patient. Let the probability of committing a type I error be 0.05 .

1-what is the assumption?

## 2-Hypothesis is?

## 3-Test statistic $=$

## 4-Reject $\mathrm{H}_{0}$ if

## 5-conclusion is:

a) reject $\mathbf{H}_{0}$

## b) accept $\mathbf{H}_{0}$

Q2:In a sample of 49 adolescents who served as the subjects in an immunologic study, one variable of interest was the diameter of a skin test reaction to an antigen. The sample mean and standard deviation were 21 and 11 mm erythematic, respectively. Can it be concluded from these data that the population mean is less than 30 ? let $\alpha=0.05$

1-what is the assumption?

## 2-Hypothesis is?

## 3-Test statistic=

## 4-Reject $\mathrm{H}_{0}$ if

## 5-conclusion is:

a)reject $\mathbf{H}_{0}$
b) accept $\mathrm{H}_{0}$

Q3:A survey of 100 similar-sized hospitals revealed a mean daily census in the pediatrics service of 27 . The population distributed normally with standard deviation of 6.5 .Do these data provide sufficient evidence to indicate that the population mean is not equal 25 ?let $\alpha=0.05$

1-what is the assumption?

## 2-Hypothesis is?

## 3-Test statistic=

4-Rejection region is

5-conclusion is:
a)reject $\mathbf{H}_{\mathbf{0}}$
b) accept $\mathrm{H}_{0}$

6- $\mathbf{P}$-value =

## H.W 1:

A research team is willing to assume that systolic blood pressures in a certain population of males are approximately normally distributed with a standard deviation of 16 . A simple random sample of 64 males from the population had a mean systolic blood pressure reading of 133 . At the 0.05 level of significance, do these data provide sufficient evidence for us to conclude that the population mean is greater than 130.

## 1-what is the assumption?

## 2-Hypothesis is?

## 3-Test statistic=

## 4-Reject $\mathrm{H}_{0}$ if

## 5-conclusion is:

a)reject $\mathrm{H}_{0} \quad$ b)accept $\mathrm{H}_{0}$
(Answer: Normal , $\sigma$ known, $\mathbf{n}$ large )
(Answer: $H_{0}: \mu \leq 130, \quad H_{A}: \mu>130$ )
(Answer: $Z=1.5$ )
(Answer: $\mathrm{Z}>\mathrm{Z}_{1-\alpha}$ )

Q4:The objective of a study by Sairam et al. (A-8) was to identify the role of various disease states and additional risk factors in the development of thrombosis. One focus of the study was to determine if there were differing levels of the anticardiolipin antibody IgG in subjects with and without thrombosis.

| Group | Mean IgG Level <br> (ml/unit) | Sample Size | Population Standard <br> deviation |
| :---: | :---: | :---: | :---: |
| Thrombosis | 59.01 | 53 | 44.89 |
| No thrombosis | 46.61 | 54 | 34.85 |

We wish to know if we may conclude, on the basis of these results, that, in general, persons with thrombosis have, on the average, higher IgG levels than persons without thrombosis. let $\alpha=0.01$

1-what is the assumption?

## 2-Hypothesis is?

## 3-Test statistic=

## 4-Acceptance region is?

## 5-conclusion is:

a)reject $\mathrm{H}_{0}$
b)accept $\mathrm{H}_{0}$

Q5: A test designed to measure mothers' attitudes toward their labor and delivery experiences was given to two groups of new mothers. Sample 1 (attenders) had attended prenatal classes held at the local health department. Sample 2 (nonattenders) did not attend the classes. The sample sizes and means and standard deviations of the test scores were as follows:

| sample | n | $\bar{x}$ | S |
| :---: | :---: | :---: | :---: |
| 1 | 15 | 4.75 | 1.0 |
| 2 | 22 | 3.00 | 1.5 |

Assume equal variances. Do these data provide sufficient evidence to indicate that attenders, on the average, score less than non attenders? Let $\alpha=0.05$.Assume normal population

1-what is the assumption?

2-Hypothesis is?

## 3- find pooled variance

## 4-Test statistic=

## 5-Reject $\mathrm{H}_{0}$ if

## 6-conclusion is:

a)reject $\mathrm{H}_{0}$
b) accept $\mathrm{H}_{0}$

## H.W 2:

Cortisol level determinations were made on two samples of women at childbirth. Group 1 subjects underwent emergency cesarean section following induced labor. Group 2 subjects delivered by either cesarean section or the vaginal route following spontaneous labor. The sample sizes, mean cortisol levels, and standard deviations were as follows:

| sample | n | $\bar{x}$ | S |
| :---: | :---: | :---: | :---: |
| 1 | 10 | 435 | 65 |
| 2 | 12 | 645 | 80 |

Assume equal variances . Do these data provide sufficient evidence to indicate a difference in the mean cortisol levels in the populations represented? Let $\alpha=0.05$, Assume normal populations

## 1-what is the assumption?

(Answer: Normal , $\sigma_{1}, \sigma_{2}$ unknown , $\mathbf{n}_{1}, \mathbf{n}_{2}$ small)

## 2-Hypothesis is?

(Answer: $\mathrm{H}_{0}: \mu_{1}=\mu_{2}, \quad \mathrm{H}_{\mathrm{A}}: \mu_{1} \neq \mu_{2}$ )

## 3- find pooled variance

(Answer: $\boldsymbol{S}_{P}^{2}=\mathbf{5 3 3 3 . 8}$ )

## 4-Test statistic=

(Answer: $\mathbf{T}=\mathbf{- 6 . 7 1 6}$ )

## 5-Acceptance Region is

(Answer: (-2.086, 2.086 )

## 6-conclusion is:

a) reject $\mathrm{H}_{0} \quad$ b)accept $\mathrm{H}_{0}$

## Q6:

Woo and McKenna (A-18) investigated the effect of broadband ultraviolet B (UVB) therapy and topical calcipotriol cream used together on areas of psoriasis. One of the outcome variables is the Psoriasis Area and Severity Index (PASI). The following table gives the PASI scores for 20 subjects measured at baseline and after eight treatments. Do these data provide sufficient evidence, at the .01 level of significance, to indicate that the combination therapy reduces PASI scores?

| subject | Baseline | After 8 treatments |
| :---: | :---: | :---: |
| 1 | 5.9 | 5.2 |
| 2 | 7.6 | 12.2 |
| 3 | 12.8 | 4.6 |
| 4 | 16.5 | 4.0 |
| 5 | 6.1 | 0.4 |
| 6 | 14.4 | 3.8 |
| 7 | 6.6 | 1.2 |
| 8 | 5.4 | 3.1 |
| 9 | 9.6 | 3.5 |
| 10 | 11.6 | 4.9 |
| 11 | 11.1 | 11.1 |
| 12 | 15.6 | 8.4 |
| 13 | 6.9 | 5.8 |
| 14 | 15.2 | 5.0 |
| 15 | 21.0 | 6.4 |
| 16 | 5.9 | 0.0 |
| 17 | 10.0 | 2.7 |
| 18 | 12.2 | 5.1 |
| 19 | 20.2 | 4.8 |
| 20 | 6.2 | 4.2 |

1 -what is the assumption?

2-Hypothesis is?

## 3-Test statistic=

## 4-Rejection region is?

## 5-conclusion is:

a) reject $\mathrm{H} 0 \quad$ b)accept H 0

## H.W3

One of the purposes of an investigation by Porcellini et al. (A-19) was to investigate the effect on CD4 T cell count of administration of intermittent interleukin (IL-2) in addition to highly active antiretroviral therapy (HAART). The following table shows the CD4 T cell count at baseline and then again after 12 months of HAART therapy with IL-2. Do the data show, at the .05 level, a significant change in CD4 T cell count?

| Subject | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CD4 T cell <br> count at entry | 173 | 58 | 103 | 181 | 105 | 301 | 169 |
| CD4 T cell <br> count at end of <br> follow-up | 257 | 108 | 315 | 362 | 141 | 549 | 369 |

1-what is the assumption?

2-Hypothesis is?

3-Test statistic=

4-Rejection region is?

5-conclusion is:
a)reject $\mathrm{H} 0 \quad$ b)accept H 0

Q7: Jacquemyn et al. (A-21) conducted a survey among gynecologists-obstetricians in the Flanders region and obtained 295 responses. Of those responding, 90 indicated that they had performed at least one cesarean section on demand every year. Does this study provide sufficient evidence for us to conclude that less than 35 percent of the gynecologistsobstetricians in the Flanders region perform at least one cesarean section on demand each year? Let $\alpha=0.05$.

## 1-Hypothesis is?

## 2-Test statistic=

## 3-Rejection region is

4-conclusion is:

$$
\text { a)reject } \mathrm{H}_{0} \quad \text { b)accept } \mathbf{H}_{\mathbf{0}}
$$

6- P -value =

## H.W4

In an article in the journal Health and Place, Hui and Bell (A-22) found that among 2428 boys ages 7 to 12 years, 461 were overweight or obese. On the basis of this study, can we conclude that more than 15 percent of the boys ages 7 to 12 in the sampled population are obese or overweight? Let $\alpha=0.05$

## 1-Hypothesis is?

2-Test statistic=
3-Acceptance region is
(Answer : Z = 4.91)
(Answer : ( $-\infty$, 1.645) )

## 4-conclusion is:

a)reject $\mathrm{H}_{0}$
b) accept $\mathbf{H}_{\mathbf{0}}$

## Q8:

Ho et al. (A-25) used telephone interviews of randomly selected respondents in Hong Kong to obtain information regarding individuals' perceptions of health and smoking history. Among 1222 current male smokers, 72 reported that they had "poor" or "very poor" health, while 30 among 282 former male smokers reported that they had "poor" or "very poor" health. Is this sufficient evidence to allow one to conclude that among Hong Kong men there is a difference between current and former smokers with respect to the proportion who perceive themselves as having "poor" and "very poor" health? Let $\alpha=0.01$.

## 1-Hypothesis is?

## 2-Test statistic=

## 3-Acceptance region is?

## 6-conclusion is:

a)reject $\mathrm{H}_{0}$
b)accept $\mathrm{H}_{0}$

## H.W5:

In a study of obesity the following results were obtained from samples of males and females between the ages of 20 and 75:

|  | n | Number overweight |
| :---: | :---: | :---: |
| Males | $\mathbf{1 5 0}$ | $\mathbf{2 1}$ |
| Females | $\mathbf{2 0 0}$ | $\mathbf{4 8}$ |

Can we conclude from these data that in the sampled populations there is a difference in the proportions who are overweight? Let $\alpha=0.05$.

## 1-Hypothesis is?

$$
\mathbf{H}_{0}: \mathbf{P}_{1}=\mathbf{P}_{2}, \quad \mathbf{H}_{A}: \mathbf{P}_{1} \neq \mathbf{P}_{2}
$$

## 2-Test statistic=

$$
Z=-2.328
$$

## 3-Acceptance region is?

(-1.645 , 1.645)

## 6-conclusion is:

a)reject $\mathrm{H}_{0}$
b)accept $\mathrm{H}_{0}$

| I | －0．09 | －0．08 | －0．07 | －0．06 | －0．05 | －0．04 | －0．03 | －0．02 | －0．01 | －0．00 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| －3．50 | 0.00017 | 0.00017 | 0.00018 | 0.00019 | 0.00019 | 0.00020 | 0.00021 | 0.00022 | 0.00022 | 0.00023 | －3．50 |
| －3 | 0.00024 | 0.0002 | 0.00026 | 0.0002 | 0.00 | 0.00029 | 0.00030 | 0.0 | 0.00032 | 0.00034 | －3．40 |
| －3．30 | 0.00035 | 0.00036 | 0.00038 | 0.00039 | 0.0004 | 0.00042 | 0.0004 | 0.00045 | 0.00047 | 0.00048 | －3．30 |
| －3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.00064 | 0.0 | 0.0 | －3．20 |
| －3 | 0.00071 | 0.00074 | 0.00076 | 0.00079 | 0.0008 | 0.0008 | 0.0008 | 0.00090 | 0.00094 | 0.00097 |  |
| －3．00 | 0.00 | 0.001 | 0.00107 | 0.0 | 0.00 | 0.00118 | 0.0012 | 0．0012 | 0.00131 | 0.0013 |  |
| －2． | 0.00139 | 0.00 | 0.00149 | 0.0 | 0.0 | 0.0 | 0.0 | 0.00175 | 0.00 | 0.00187 |  |
| －2 | 0.0 | 0.0 | 0.00205 | 0.0 | 0.0 | 0.00226 | 0.0 | 0.00240 | 0.00248 | 0.00256 | －2．80 |
| －2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.00307 | 0.00 | 0.00326 | 0.00336 | 0.00347 | －2．70 |
| －2．60 | 0.00357 | 0.00368 | 0.00379 | 0.003 | 0.00 | 0.00 | 0.00 | 0.00440 | 0.00453 | 0.00466 | －2．60 |
| －2．50 | 0. | 0.0 | 0.00508 | 0.00523 | 0. | 0. | 0.00570 | 0.00587 | 0.00604 | 0.00621 | －2．50 |
| －2．40 | 0.00639 | 0.0065 | 0.00676 | 0.0069 | 0.0 | 0.00734 | 0.00 | 0.00776 | 0.00798 | 0.00820 | －2．40 |
| －2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. | 0.01044 | 0.01072 | －2．30 |
| －2． | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0． | 0.0 | 0.01390 | －2．20 |
| －2．10 | 0.01426 | 0.01463 | 0.01500 | 0.01539 | 0.01578 | 0.01618 | 0.01659 | 0.01700 | 0.01743 | 0.01786 |  |
| －2．00 | 0.0 | 0. | 0.0 | 0.0 | 0.0 | 0.02068 | 0.0 | 0.02169 | 0.02222 | 0.02275 | －2．00 |
| －1．9 | 0.02330 | 0.0238 | 0.02442 | 0.02500 | 0.0255 | 0.02619 | 0.02680 | 0.02743 | 0.02807 | 0.02872 | －1．90 |
| －1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. | 0.03515 | 0.0 |  |
| －1．70 | 0.03673 | 0.0375 | 0.03836 | 0.03920 | 0.04 | 0.04093 | 0.04182 | 0.04272 | 0.04363 | 0.04457 | － |
| －1．60 | 0.0 | 0.04 | 0.04 | 0.048 | 0.04 | 0.05050 | 0.0 | 0.05262 | 0.0 | 0.0 |  |
| －1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. | 2 | 0.0 | －1．50 |
| －1．40 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.07780 | 0.0 | 0.08076 |  |
| －1 | 0.08 | 0.08 | 0.0 | 0.0 | 0.0 | 0.09012 | 0.0 | 0.09342 | 0. | 0.0 |  |
| －1．20 | 0.09853 | 0.10027 | 0.10204 | 0.10383 | 0.10565 | 0.10749 | 0.10935 | 0.11123 | 0.11314 | 0.11507 | － |
| －1．10 | 0.11702 | 0.11 | 0.12100 | 0.1 | 0.1 | 0.12714 | 0.1 | 0.13136 | 0.13350 | 0.13567 | －1．10 |
| －1． | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.14917 | 0.1 | 0.15386 | 0.15625 | 0.1 |  |
| －0．90 | 0.16109 | 0.1 | 0.16 | 0.1 | 0.1 | 0.1 | 0.1 | 0.17879 | 0.18 | 0.18406 |  |
| －0．80 | 0.18 | 0.18 | 0.19 | 0.19 | 0.1 | 0.20045 | 0.2032 | 0.20611 | 0.20897 | 0.21186 | －0．80 |
| －0．70 | 0.21476 | 0.2 | 0.22065 | 0.2 | 0.2 | 0.2 | 0.2 | 0.23576 | 0.2 | 0.24196 | －0．70 |
| －0．60 | 0.24510 | 0.2482 | 0.25143 | 0.25 | 0.25 | 0.26109 | 0.264 | 0.26763 | 0.27093 | 0.27425 | －0．60 |
| －0， | 0.2 | 0.28 | 0.2 | 0.2 | 0.2 | 0.2 | 0.29806 | 0.30153 | 0.30503 | 0.30854 | $-0.50$ |
| －0．4 | 0.31207 | 0.31561 | 0.31918 | 0.32276 | 0.32 | 0.32997 | 0.33360 | 0.33724 | 0.3409 | 0.34458 | －0．40 |
| －0．30 | 0.34 | 0.35 | 0.35569 | 0.3 | 0.3 | 0.3669 | 0.37070 | 0.37448 | 0.37828 | 0.38209 | －0．30 |
| －0．20 | 0.38591 | 0.38974 | 0.39358 | 0.39743 | 0.4012 | 0.40517 | 0.40905 | 0.41294 | 0.41683 | 0.42074 | $-0.20$ |
| －0．10 | 0.42465 | 0.42858 | 0.43251 | 0.43644 | 0.44038 | 0.44433 | 0.44828 | 0.45224 | 0.45620 | 0.46017 | －0．10 |
| －0．00 | 0.46414 | 0.46812 | 0.47210 | 0.47608 | 0.48006 | 0.48405 | 0.48803 | 0.49202 | 0.49601 | 0.50000 | －0．0 |


| 0 S ¢ | £86660 | \＆86660 | 286660 | 186660 | 186660 | 086660 | $6 L 6660$ | 826660 | 826660 | LL6660 | ＇E |
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| 08 | S96660 | t96660 | 296660 | 196660 | 096660 | 856660 | LS6660 | SS6660 | \＆S6660 | 2S6660 | \％$\varepsilon$ |
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| 0 L | 676660 | 976660 | t76660 | I26660 | 816660 | 916660 | \＆I6660 | 016660 | 906660 | \＆06660 | 01＇$¢$ |
| $00{ }^{\circ} \mathrm{E}$ | 06660 | 968660 | E68660 | 688660 | 988660 | 78866 | 8L866 | t 28660 | 698660 | S98660 | $00^{\circ} \varepsilon$ |
| $06 \%$ | I9 | 958 | IS8660 | 9t | It | 988660 | IE8660 | S78660 | 618660 | \＆18660 | $06^{\prime}$ |
| 08 | 108660 | I08660 | S6L660 | 882660 | I8L660 | tLL660 | L9L660 | 09L660 | ZSL660 | ttL660 | $08^{\prime} \tau$ |
| 0L＇z | 9EL66 | 87L660 | 0ZL66 | II | 20 | \＆69 | \＆89660 | $t \angle 9$ | t99660 | \＆S9660 | $0 L^{\circ} \mathrm{z}$ |
| $09^{\prime} \tau$ | Et9 | て£96 | 179660 | 609 | 86S6 | S8 | EL | 09S | LtS660 | tE | 09＇z |
| 0S゙て | 025660 | 90¢660 | $26+660$ | LLt66 | $19+660$ | 9tt660 | 0\＆t660 | \＆1t660 | 968660 | $6 L$ 6660 | $0 ¢ 7$ |
| $00^{\prime}$ | 19866 | と | たた | S0E | 987660 | 99 | Sた6 | たて660 | 202660 | 08160 | $0 \dagger^{\prime} \tau$ |
|  | 8SI66 | †¢I66 | III | 98066 | 19066 | 9 9066 | 0I066 | \＆86860 | 9¢6860 | 876860 | $00^{\circ} \mathrm{z}$ |
| 0で | 66886 | 028860 | $0+886$ | 60886 | 8LL86 | StL86 | \＆IL860 | $6 L 9860$ | St9860 | 019860 | $00^{\circ}$ |
| 01＇z | tLS8 | L®S | 00s | 19t | 27t8 | 288 | It | $00 \varepsilon 860$ | Ls | V1880 | 01＇z |
| $00 \%$ | 691860 | 七てI860 | LL0860 | 0ع0860 | 286L60 | 286L60 | 288L60 | IE8L60 | 8LLL60 | SZLL60 | $00^{\circ}$ |
| 06＇I | 0L9L6 | SI | 85 | 00 | It | I8 | 0z\＆ | LS | E6 | 8 | 06＇I |
| $08^{\prime}$ | 290L6 | S6696 | 976 | 958 | t8L9 | ZIL | 8\＆ | 29S | S8t | LOt | 08＇I |
|  | LZE960 | 9t2960 | t91960 | 08096 | t66560 | L06S60 | 818560 | 8ZLS60 | LE9S60 | \＆tSS60 | 0L＇I |
| 09 I | 6tt | ZSES6 | ts | tS | \＆SO | OS | St | 8\＆ | 0ع | 02 | 09＇I |
| 0 S | 80tt60 | S67t60 | 6L1660 | 290t6 | Et686 | 27886 | 6698 | tLS\＆60 | 8tte60 | 6Iを\＆60 | 0S＇I |
| $0 \pm$ | 681860 | 950ع60 | 276760 | S8L26 | Lt9760 | LOS760 | t9\＆260 | 077760 | ELO260 | t26160 | T |
| $0 \mathrm{E}^{\prime} \mathrm{I}$ | tLLI6 | I2916 | 99t | 60\＆1 | 6t | 886 | t 28 | 859 | 06t | $07 \varepsilon$ | $0 \varepsilon^{\prime}$＇I |
|  | LtI060 | EL6680 | 96L680 | LI | SEt680 | IS7 | S90 | LL8880 | 989880 | \＆6t88 0 | ＇I |
| 0I＇I | $86788^{\circ}$ | 00I880 | 006L8 0 | 869 L8 | \＆6t $\angle 8$ | 98ZL80 | 9L0L8 | t98980 | 0S998 0 | \＆$¢ 598^{\circ} 0$ | 0I＇I |
| $00 \cdot \mathrm{I}$ | tI298 | \＆6658 | 69L | EtSS8 | tits | \＆80 | 6 t | 七I9 | SLEt80 | † | $00^{\circ} \mathrm{I}$ |
| $06^{\prime} 0$ | I68 | 9ts | 868 | Lt | t6 | $6 \varepsilon$ | 18 | I2 | $65818^{\circ} 0$ | 十6SI8 0 | $06^{\circ} 0$ |
| 08＇0 | LZ¢18 | LSOI8 | S8LO | IISO | † 2708 | SS66 | EL9 | 6886 | E0I6LO | bI88L0 | ＇0 |
| 020 | tてS8LO | 0عZ8L＇0 | Sc6LLO | LE9LLO | L\＆\＆LLO | SEOLLO | OELOLO | tてt9LO | SIITCO | t08SLO | 0 |
| $00^{\prime} 0$ | 06 | SL | LS | Lع | SI | I68EL＇0 | S9S | LعZEL＇0 | L062LO | SLSCL＇0 | 09＇0 |
| 0S＂0 | 0tzZLio | t06IL＇0 | 99SILO | 97ZIL | t880LO | OtSOLO | 7610LO | Lt869 0 | L6t69 0 | 9tI69 0 | 0 |
| Ob0 | E6L890 | 6\＆t890 | 280890 | tiLL | t9EL90 | E00L90 | 0t999 | 9L299 0 | 016S9 0 | 2tSS9 0 | 0 |
| 08＇0 | ELIS9 0 | ع08t9 ${ }^{\circ}$ | İtt9 0 | 850t9 ${ }^{\circ}$ | \＆89を90 | LO\＆と90 | $08629^{\circ}$ | 2S¢790 | ZLIZ9 0 | I6LI9 0 | 0 |
| 0で0 | 60tI9 0 | 920190 | 2t909 0 | LSZ09 0 | IL865 0 | E8t650 | S60650 | 90L8S0 | LIE850 | 976LS 0 | 0z＇0 |
| $0{ }^{\text {［ }} 0$ | SESLSO | てtILSO | 6tL9 0 | 9SE9S0 | 296SS0 | L9SSSO | ZLISS 0 | 9LLtSO | 08\＆tS 0 | \＆86¢S0 | $0{ }^{0} 0$ |
| $00^{\circ} 0$ | 98S\＆S＇0 | 88IES＇0 | 06LZS0 | 26E2SO | t66IS＇0 | S6SISO | L6IISO | 86LOSO | 6680S0 | 000050 | $00^{\circ} 0$ |
| z | $60^{\circ} 0$ | $80^{\circ} 0$ | L0＇0 | $90^{\circ} 0$ | S00 | t000 | E0＇0 | $20{ }^{\circ} 0$ | ［0＇0 | $00{ }^{\circ}$ | z |

Critical Values of the $t$-distribution $\left(t_{\alpha}\right)$


| $\boldsymbol{v}=\mathbf{d f}$ | $\mathbf{t}_{0.90}$ | $\mathbf{t}_{0.95}$ | $\mathbf{t}_{0.975}$ | $\mathbf{t}_{0.99}$ | $\mathbf{t}_{0.995}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 35 | 1.3062 | 1.6896 | 2.0301 | 2.4377 | 2.7238 |
| 40 | 1.3030 | 1.6840 | 2.0210 | 2.4230 | 2.7040 |
| 45 | 1.3006 | 1.6794 | 2.0141 | 2.4121 | 2.6896 |
| 50 | 1.2987 | 1.6759 | 2.0086 | 2.4033 | 2.6778 |
| 60 | 1.2958 | 1.6706 | 2.0003 | 2.3901 | 2.6603 |
| 70 | 1.2938 | 1.6669 | 1.9944 | 2.3808 | 2.6479 |
| 80 | 1.2922 | 1.6641 | 1.9901 | 2.3739 | 2.6387 |
| 90 | 1.2910 | 1.6620 | 1.9867 | 2.3685 | 2.6316 |
| 100 | 1.2901 | 1.6602 | 1.9840 | 2.3642 | 2.6259 |
| 120 | 1.2886 | 1.6577 | 1.9799 | 2.3578 | 2.6174 |
| 140 | 1.2876 | 1.6558 | 1.9771 | 2.3533 | 2.6114 |
| 160 | 1.2869 | 1.6544 | 1.9749 | 2.3499 | 2.6069 |
| 180 | 1.2863 | 1.6534 | 1.9732 | 2.3472 | 2.6034 |
| 200 | 1.2858 | 1.6525 | 1.9719 | 2.3451 | 2.6006 |
| $\infty$ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |

