Exercises

Q1 Suppose that you are given observations y_1 and y_2 such that: $y_1 = \alpha + \beta + \epsilon_1$ $y_2 = -\alpha + \beta + \epsilon_2$ The random variables ϵ_i, for $i = 1, 2$, are independent and normally distributed with mean 0 and variance σ^2.

(a) Find the least squares estimators of the parameters α and β, and verify that they are unbiased estimators. Hint: obtain the minimum of the sum of the ϵ_i^2 using the least squares technique.

Q2 An investigation, conducted by a mail-order company, into the relationship between the sales revenues (y_i, in millions of dollars) and the price per gallon of gasoline (x_i, in cents) over a period of 10 months yields:

$\sum_{i=1}^{10} y_i = 527, \sum_{i=1}^{10} x_i = 6509, \sum_{i=1}^{10} x_i^2 = 4909311 and \sum_{i=1}^{10} x_i y_i = 325243.$

Estimate the parameters β_0 and β_1 in the regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, where the ϵ_i are uncorrelated with a mean of zero and a common variance of σ^2 for i = 1, . . . , 10. Interpret the estimated regression line.

Q3 Assuming a regression line fitted using LSE, show that

(a)$\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \hat{y}_i$

(b)$\sum_{i=1}^{n} x_i e_i = 0$

(c)$\sum_{i=1}^{n} \hat{y}_i e_i = 0$.

Q4 Let X and ϵ be two independent random variables, and assume $E(\epsilon) = 0$. Let $Y = \beta_0 + \beta_1 X + \epsilon$. Show that: $\beta_1 = \frac{Cov(X,Y)}{V(X)} = Corr(X,Y)\sqrt{\frac{V(Y)}{V(X)}}$