

# Exercises

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.

b) Miami is the capital of Florida.

c)  $2 + 3 = 5$ .

d)  $5 + 7 = 10$ .

e)  $x + 2 = 11$ .

2. Which of these are propositions? What are the truth values of those that are propositions?

a) Do not pass go.

b) What time is it?

c) There are no black flies in Maine.

d)  $4 + x = 5$ .

e) The moon is made of green cheese.

f)  $2^n \geq 100$ .

3. What is the negation of each of these propositions?

a) Today is Thursday.

1. Randomly 12

b) There is no pollution in New Jersey.

c)  $2 + 1 \neq 3$ .

d) The summer in Maine is hot and sunny.

Let  $p$  and  $q$  be the propositions

$p$ : I bought a lottery ticket this week

$q$ : I won the million dollar jackpot on Friday.

Express each of these propositions as an English sentence

a)  $\neg p \wedge q$

b)  $p \vee q$

c)  $p \leftrightarrow q$

d)  $p \wedge q$

e)  $\neg p \wedge \neg q$

f)  $\neg p \vee (p \wedge q)$

5. Let  $p$  and  $q$  be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

a)  $\neg q$

b)  $p \wedge q$

c)  $\neg p \vee q$

d)  $p \rightarrow \neg q$

e)  $\neg q \rightarrow p$

f)  $\neg p \rightarrow \neg q$

g)  $p \leftrightarrow \neg q$

h)  $\neg p \wedge (p \vee \neg q)$

Links



المسحور الحقيقى

ما هو نفي كل  
نص من النصا حذره  
تلقا

- a)  $p \rightarrow \neg p$
- b)  $(p \vee \neg r) \wedge (q \vee \neg s)$
- c)  $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$
- d)  $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$

26. How many rows appear in a truth table for each of these compound propositions?

- a)  $(q \rightarrow \neg p) \vee (\neg p \rightarrow \neg q)$
- b)  $(p \vee \neg t) \wedge (p \vee \neg s)$
- c)  $(p \rightarrow r) \vee (\neg s \rightarrow \neg t) \vee (\neg u \rightarrow v)$
- d)  $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

27. Construct a truth table for each of these compound propositions.

- a)  $p \wedge \neg p$
- b)  $p \vee \neg p$
- c)  $(p \vee \neg q) \rightarrow q$
- d)  $(p \vee q) \rightarrow (p \wedge q)$
- e)  $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
- f)  $(p \rightarrow q) \rightarrow (q \rightarrow p)$

28. Construct a truth table for each of these compound propositions.

- a)  $p \rightarrow \neg p$
- b)  $p \leftrightarrow \neg p$
- c)  $p \oplus (p \vee q)$
- d)  $(p \wedge q) \rightarrow (p \vee q)$
- e)  $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
- f)  $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

29. Construct a truth table for each of these compound propositions.

- a)  $(p \vee q) \rightarrow (p \oplus q)$
- b)  $(p \oplus q) \rightarrow (p \wedge q)$
- c)  $(p \vee q) \oplus (p \wedge q)$
- d)  $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
- e)  $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
- f)  $(p \oplus q) \rightarrow (p \oplus \neg q)$

30. Construct a truth table for each of these compound propositions.

- a)  $p \oplus p$
- b)  $p \oplus \neg p$
- c)  $p \oplus \neg q$
- d)  $\neg p \oplus \neg q$
- e)  $(p \oplus q) \vee (p \oplus \neg q)$
- f)  $(p \oplus q) \wedge (p \oplus \neg q)$

31. Construct a truth table for each of these compound propositions.

- a)  $p \rightarrow \neg q$
- b)  $\neg p \leftrightarrow q$
- c)  $(p \rightarrow q) \vee (\neg p \rightarrow q)$
- d)  $(p \rightarrow q) \wedge (\neg p \rightarrow q)$
- e)  $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
- f)  $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

32. Construct a truth table for each of these compound propositions.

- a)  $(p \vee q) \vee r$
- b)  $(p \vee q) \wedge r$
- c)  $(p \wedge q) \vee r$
- d)  $(p \wedge q) \wedge r$
- e)  $(p \vee q) \wedge \neg r$
- f)  $(p \wedge q) \vee \neg r$

33. Construct a truth table for each of these compound propositions.

- a)  $p \rightarrow (\neg q \vee r)$
- b)  $\neg p \rightarrow (q \rightarrow r)$
- c)  $(p \rightarrow q) \vee (\neg p \rightarrow r)$
- d)  $(p \rightarrow q) \wedge (\neg p \rightarrow r)$
- e)  $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
- f)  $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

34. Construct a truth table for  $((p \rightarrow q) \rightarrow r) \rightarrow s$ .

35. Construct a truth table for  $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$ .

36. What is the value of  $x$  after each of these statements is encountered in a computer program, if  $x = 1$  before the statement is reached?

- a) if  $1 + 2 = 3$  then  $x := x + 1$
- b) if  $(1 + 1 = 3)$  OR  $(2 + 2 = 3)$  then  $x := x + 1$
- c) if  $(2 + 3 = 5)$  AND  $(3 + 4 = 7)$  then  $x := x + 1$
- d) if  $(1 + 1 = 2)$  XOR  $(1 + 2 = 3)$  then  $x := x + 1$
- e) if  $x < 2$  then  $x := x + 1$

37. Find the bitwise OR, bitwise AND, and bitwise XOR of each of these pairs of bit strings.

- a) 101 1110, 010 0001
- b) 1111 0000, 1010 1010
- c) 00 0111 0001, 10 0100 1000
- d) 11 1111 1111, 00 0000 0000

38. Evaluate each of these expressions.

- a)  $1\ 1000 \wedge (0\ 1011 \vee 1\ 1011)$
- b)  $(0\ 1111 \wedge 1\ 0101) \vee 0\ 1000$
- c)  $(0\ 1010 \oplus 1\ 1011) \oplus 0\ 1000$
- d)  $(1\ 1011 \vee 0\ 1010) \wedge (1\ 0001 \vee 1\ 1011)$

Fuzzy logic is used in artificial intelligence. In fuzzy logic, a proposition has a truth value that is a number between 0 and 1, inclusive. A proposition with a truth value of 0 is false and one with a truth value of 1 is true. Truth values that are between 0 and 1 indicate varying degrees of truth. For instance, the truth value 0.8 can be assigned to the statement "Fred is happy," because Fred is happy most of the time, and the truth value 0.4 can be assigned to the statement "John is happy," because John is happy slightly less than half the time.

39. The truth value of the negation of a proposition in fuzzy logic is 1 minus the truth value of the proposition. What are the truth values of the statements "Fred is not happy" and "John is not happy"?

40. The truth value of the conjunction of two propositions in fuzzy logic is the minimum of the truth values of the two propositions. What are the truth values of the statements "Fred and John are happy" and "Neither Fred nor John is happy"?

41. The truth value of the disjunction of two propositions in fuzzy logic is the maximum of the truth values of the two propositions. What are the truth values of the statements "Fred is happy, or John is happy" and "Fred is not happy or John is not happy"?

\*42. Is the assertion "This statement is false" a proposition?

\*43. The  $n$ th statement in a list of 100 statements is "Exactly  $n$  of the statements in this list are false."

- a) What conclusions can you draw from these statements?
- b) Answer part (a) if the  $n$ th statement is "At least  $n$  of the statements in this list are false."
- c) Answer part (b) assuming that the list contains 99 statements.

44. An ancient Sicilian legend says that the barber in a remote town who can be reached only by traveling a dangerous mountain road shaves those people, and only those