Chapter 5: General Probability

Q1. Suppose that a fair die is thrown twice independently, then

1. the probability that the sum of numbers of the two dice is less than or equal to 4 is;
   (A) 0.1667 (B) 0.6667 (C) 0.8333 (D) 0.1389
2. the probability that at least one of the die shows 4 is;
   (A) 0.6667 (B) 0.3056 (C) 0.8333 (D) 0.1389
3. the probability that one die shows one and the sum of the two dice is four is;
   (A) 0.0556 (B) 0.6667 (C) 0.3056 (D) 0.1389
4. the event A={the sum of two dice is 4} and the event B={exactly one die shows two} are,
   (A) Independent (B) Dependent (C) Joint (D) None of these.

Solution of :

$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$

$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$

$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$

$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$

$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$

$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

1- A: *the sum of numbers of the two dice* ≤ 4

$$A = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\} \to P(A) = \frac{6}{36} = \frac{1}{6} = 0.1667$$

2- B: *at least one of the die shows* 4

$$B = \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4),(4,1),(4,2),(4,3),(4,5),(4,6)\}$$

$$P(B) = \frac{11}{36} = 0.3056$$

3- C: *that one die shows* (1) *and the sum of the two dice* $= 4$

$$C = \{(1,3),(3,1)\} \to P(C) = \frac{2}{36} = 0.0556$$

4- $D: the\ sum\ of\ two\ dice = 4\ , D = \{(1,3),(3,1),(2,2)\} \rightarrow P(D) = \frac{3}{36} = 0.0833$

$E: one\ die\ shows\ 2$

$E = \{(1,2),(2,1),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$

$P(E) = \frac{10}{36} = 0.2778$

$D \cap E = \{\ \} \rightarrow P(D \cap E) = 0\ \therefore D\ and\ E\ are\ disjoint.$

$P(D \cap E) = 0 \neq P(D)P(E) \rightarrow D\ and\ E\ are\ dependent.$

Q2. The probability that a computer system has an electrical failure is 0.15, and the probability that it has a virus is 0.25, and the probability that it has both problems is 0.10, then the

a) probability that the computer system has the electrical failure, or the virus is:

(A) 1.15 (B) 0.2 (C) 0.15 (D) 0.30

Solution :

Event A: The computer system has an electrical failure

Event B: The computer has a virus

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.15 + 0.25 - 0.1 = 0.3$

b) the event A: {The computer system has an electrical failure } and the event B: { The computer has a virus } are,

(A) Exhaustive (B) disjoint (Mutually exclusive)

(C) Independent (D) Dependent

Exhaustive:

Is $A \cup B = \Omega \equiv P(A \cup B) = 1$?

$P(A \cup B) = 0.3 \neq 1$ then they are not Exhaustive

disjoint (Mutually exclusive):

Is $A \cap B = \emptyset \equiv P(A \cap B) = 0$

$P(A \cap B) = 0.1 \neq 0$ then they are not disjoint (Mutually exclusive)

Independent:

Is $P(A \cap B) = P(A)P(B) \equiv P(A|B) = P(A) \equiv P(B|A) = P(B)$?

since $0.1 \neq 0.15(0.25)$ then they are <u>not Independent (dependent)</u>

Q3. If the probability of passing course (A) is 0.6, passing course (B) is 0.7, passing course A or B is 0.9. Find:

1- Probability of passing course A and B.
2- Probability of passing course A only.
3- Probability of passing course B and not passing course A.
4- Probability of not passing course A and B.
5- Probability of passing course B or not passing course A.

Solution:

1- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Then $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.7 - 0.9 = 0.4$

2- $P(A \cap B^C) = ??$ we know that $P(A) = P(A \cap B) + P(A \cap B^C)$
Then, $P(A \cap B^C) = P(A) - P(A \cap B) = 0.6 - 0.4 = 0.2$

3- $P(B \cap A^C) = ??$ we know that $P(B) = P(B \cap A) + P(B \cap A^C)$
Then, $P(B \cap A^C) = P(B) - P(A \cap B) = 0.7 - 0.4 = 0.3$

4- $P(A \cap B)^c$ =1- $P(A \cap B) = 1 - 0.4 = 0.6$

5- $P(B \cup A^C) = P(B) + P(A^C) - P(B \cap A^C)$

$= 0.7 + 0.4 - 0.3 = 0.8$

Q4. The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$; the probability that it arrives on time is $P(A) = 0.82$; and the probability that it departs and arrives on time is $P(D \cap A) = 0.78$.
Find the probability that a plane

1- arrives on time given that it departed on time.
2- departed on time given that it has arrived on time.
3- arrived on time given that it has not departed on time.

Solution :

1- $P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{0.78}{0.83}$=0.9398

2- $P(D|A) = \frac{P(A \cap D)}{P(A)} = \frac{0.78}{0.82}$=0.9512

3- $P(A|D^C) = \frac{P(A \cap D^C)}{P(D^C)}$
We know that, $P(A) = P(A \cap D) + P(A \cap D^C)$
0.82 = 0.78 + $P(A \cap D^C)$
$P(A \cap D^C) = 0.04$

And $P(D^C) = 1 - P(D) = 1 - 0.83 = 0.17$

Then,

$$P(A|D^C) = \frac{P(A \cap D^C)}{P(D^C)} = \frac{0.04}{0.17} = 0.2353$$

Question 5: In a school, there are 60 students enrolled in various extracurricular activities. The breakdown is as follows:

- 25 students participate in the Music club (M).
- 30 students are part of the Sports team(S).
- 15 students are in the Art club (A).
- 10 students are involved in both the Music club and Sports team.
- 5 students are in both the Music club and Art club.
- 8 students are in both the Sports team and Art club.
- 3 students are in all three clubs.

What is the probability that a student chosen at random is involved in at least one of the activities?

Solution :

$$P(M \cup S \cup A) = P(M) + P(S) + P(A) - P(M \cap S) - P(M \cap A) - P(S \cap A) + P(M \cap S \cap A)$$

$$= \frac{25+30+15-10-5-8+3}{60} = \frac{50}{60} = 0.833$$

Question 6 :

1- Given $P(A \cap B) = 0:3; P(A) = 0:5; P(B) = 0:4$, *find* $P(Ac \cap Bc)$

Solution :

$P(A^c \cap B^c) = P(A \cup B)^c$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5+0.4-0.3=0.6$

Then, $P(A \cup B)^c = 1 - P(A \cup B) = 1-0.6=0.4$

$P(A^c \cap B^c) = 0.4$

2- If the events A_1, A_2 and A_3 are mutually independent, P(A_1) =0.3, P(A_2)= 0.4 and P(A_3)= 0.2 Then P($A_1 \cap A_2 \cap A_3$)=

Since the events A_1, A_2 and A_3 are mutually independent then,

P($A_1 \cap A_2 \cap A_3$)= P(A_1) P(A_2) P(A_3) =0.3(0.4)(0.2)= 0.024

Question 7: A basket A contains 20 red apples, 15 green apples. And basket B contains 14 red apples and 18 green apple. A person randomly selects one basket and take out an apple from the basket. What is the probability that the apple selected is red? (Assuming the baskets are picked with equal probability.

$P(A) = \frac{1}{2}, P(B) = \frac{1}{2} \quad P(R|A) = \frac{20}{35} \quad P(R|B) = \frac{14}{32}$

$P(R) = P(R|A)P(A) + P(R|B)P(B)$

$$= \frac{20}{35} * \frac{1}{2} + \frac{14}{32} * \frac{1}{2} = 0.5045$$

Bayes Rule

Question 8: Upon arrival at a hospital's emergency room, patients are categorized according to their condition as critical, serious, or stable. In the past year:

(i) 10% of the emergency room patients were critical;
(ii) 30% of the emergency room patients were serious;
(iii) the rest of the emergency room patients were stable;
(iv) 40% of the critical patients died;
(vi) 10% of the serious patients died; and
(vii) 1% of the stable patients died.

Given that a patient survived, calculate the probability that the patient was categorized as serious upon arrival.

(A) 0.06 (B) 0.29 (C) 0.30 (D) 0.39 (E) 0.64

Solution :

Let C=critical ; SE= serious ; ST= stable ; D= died ; SU= survive

We are given that P(C)=0.1 , P(SE)=0.3 , P(ST)=1-(0.1+0.3)=0.6 ,

P(D|C)=0.4 , P(D|SE)=0.1 , P(D|ST)=0.01

Therefore, $P(SU|SE) = 1 -$ P(D|SE) = 1- 0.1 =0.9

$$P(SE|SU) = \frac{P(SU|SE)\,P(\mathrm{SE})}{P(SU|C)\,P(C) + P(SU|SE)\,P(\mathrm{SE}) + P(SU|ST)\,P(\mathrm{ST})}$$

$$= \frac{(0.9)(0.3)}{(0.6)(0.1) + (0.9)(0.3) + (0.99)(0.6)} = 0.29$$

Question 9:. An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are:

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Young adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05
Total	100%	

Given that a driver has been involved in at least one collision in the past year, calculate the probability that the driver is a young adult driver.
(A) 0.06 (B) 0.16 (C) 0.19 (D) 0.22 (E) 0.25

Solution : H.W
Let
C = Event of a collision
T = Event of a teen driver
Y = Event of a young adult driver
M = Event of a midlife driver
S = Event of a senior driver
Then,

$$P(Y|C) = \frac{P(C|Y)\,P(\mathrm{Y})}{P(C|T)\,P(T) + P(C|Y)\,P(\mathrm{Y}) + P(C|M)\,P(\mathrm{M}) + P(C|S)\,P(\mathrm{S})}$$

$$= \frac{(0.08)(0.16)}{(0.15)(0.08) + (0.08)(0.16) + (0.04)(0.45) + (0.05)(0.31)} = 0.22$$

Question 10:. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not actually present. **<u>One percent of the population actually has the disease</u>.** Calculate the probability that a person actually has the disease given that the test indicates the presence of the disease.
(A) 0.324 (B) 0.657 (C) 0.945 (D) 0.950 (E) 0.995

Solution :

Commented [BM1]: Question 23 From (problems from SOA)
درس خبير اكتواري احتمالية تورط أنواع مختلفة من السائقين في اصطدام واحد على الأقل خلال أي فترة سنة واحدة. نتائج الدراسة هي:

Let T = positive test result
D = disease is present D^C = disease is absent $P(D) = 0.01$ $P(D^C) = 1 - P(D) = 0.99$
$P(T|D) = 0.95$ $P(T|D^C) = 0.05$
Then,

$$P(D|T) = \frac{P(T|D)\,P(D)}{P(T|D)\,P(D) + P(T|D^C)\,P(D^C)} = \frac{(0.95)(0.01)}{(0.95)(0.01) + (0.05)(0.99)} = 0.1610$$

Question 11. The probability that a randomly chosen male has a blood circulation problem is 0.25. Males who have a blood circulation problem are twice as likely to be smokers as those who do not have a blood circulation problem.
Calculate the probability that a male has a blood circulation problem, given that he is a smoker.
(A) 1/4   (B) 1/3   (C) 2/5   (D) 1/2   (E) 2/3

Commented [BM2]: Question 26 From (problems from SOA)
احتمال إصابة الذكر المختار عشوائياً بمشكلة في الدورة الدموية هو 0.25. الذكور الذين يعانون من مشاكل في الدورة الدموية هم أكثر عرضة مرتين لأن يكونوا مدخنين مثل أولئك الذين لا يعانون من مشاكل في الدورة الدموية.

Solution :
Let:
S = Event of a smoker
C = Event of a circulation problem
Then we are given that $P[C] = 0.25$ and $P[S \mid C] = 2\,P[S \mid C^C]$
Then,

$$P(C|S) = \frac{P(S|C)P(C)}{P(S|C)P(C) + P(S|C^C)P(C^C)} = \frac{2\,P(S|C^C)P(C)}{2\,P(S|C^C)P(C) + P(S|C^C)P(C^C)}$$

$$= \frac{2P(C)}{2P(C) + P(C^C)} = \frac{2\,(0.25)}{2(0.25) + 0.75} = \frac{2}{5}$$