Example 1

In a study conducted by the Department of Mechanical Engineering at Virginia Tech, the steel rods supplied by two diﬀerent companies were compared. Ten sample springs were made out of the steel rods supplied by each company, and a measure of ﬂexibility was recorded for each. The data are as follows:

|  |  |
| --- | --- |
| Company A | Company B |
| 9.3 | 11 |
| 8.8 | 9.8 |
| 6.8 | 9.9 |
| 8.7 | 10.2 |
| 8.5 | 10.1 |
| 6.7 | 9.7 |
| 8 | 11 |
| 6.5 | 11.1 |
| 9.2 | 10.2 |
| 7 | 9.6 |

Example 2

A dynamic cone penetrometer (DCP) is used for measuring material resistance to penetration (mm/blow) as a cone is driven into pavement or sub grade. Suppose that for a particular application, it is required that the true average DCP value (µ) for a certain type of pavement **be less than 30**. The pavement will not be used unless there is conclusive evidence that the specification has been met. Let’s state and test the appropriate hypotheses using the following data (“Probabilistic Model for the Analysis of Dynamic Cone Penetrometer Test Values in Pavement Structure Evaluation,” J. of Testing and Evaluation, 1999: 7–14):

14.1 14.5 15.5 16.0 16.0 16.7 16.9 17.1 17.5 17.8

17.8 18.1 18.2 18.3 18.3 19.0 19.2 19.4 20.0 20.0

20.8 20.8 21.0 21.5 23.5 27.5 27.5 28.0 28.3 30.0

30.0 31.6 31.7 31.7 32.5 33.5 33.9 35.0 35.0 35.0

36.7 40.0 40.0 41.3 41.7 47.5 50.0 51.0 51.8 54.4

55.0 57.0

Example 3

Suppose that an engineer is interested in testing the bias in a pH meter. Data are collected on a neutral substance (pH= 7.0). A sample of the measurements were taken with the data as follows:

7.07 7.00 7.10 6.97 7.00 7.03 7.01 7.01 6.98 7.08

It is, then, of interest to test $H\_{0}:μ=7$ vs $H\_{1}: μ\ne 7$

Example 4

The following data represent the running times of ﬁlms produced by two motion-picture companies.

|  |  |
| --- | --- |
| **Company** | **Time (minutes)** |
| **I** | 103 | 94 | 110 | 87 | 98 |  |  |
| **II** | 97 | 82 | 123 | 92 | 175 | 88 | 118 |

Compute a 90% conﬁdence interval for the diﬀerence between the average running times of ﬁlms produced by the two companies. Assume that the running-time differences are approximately normally distributed with **unequal variances**.

Example 5

To find out whether a new serum will arrest leukemia, 9 mice, all with an advanced stage of the disease, are selected. Five mice receive the treatment and 4 do not. Survival times, in years, from the time the experiment commenced are as follows:

Treatment 2.1 5.3 1.4 4.6 0.9

No Treatment 1.9 0.5 2.8 3.1

At the 0.05 level of significance, can the serum be said to be effective? Assume the two populations to be normally distributed with equal variances. Test $H\_{0}:μ\_{1}=μ\_{2}$ vs $H\_{1}: μ\_{1}\ne μ\_{2}$

Example 6:

A clinic provides a program to help their clients lose weight and asks a consumer agency to investigate the effectiveness of the program. The agency takes a sample of 15 people, weighing each person in the sample before the program begins and 3 months later to produce the table below

|  |  |  |
| --- | --- | --- |
| Person | Before | After |
| 1 | 210 | 197 |
| 2 | 205 | 195 |
| 3 | 193 | 191 |
| 4 | 182 | 174 |
| 5 | 259 | 236 |
| 6 | 239 | 226 |
| 7 | 164 | 157 |
| 8 | 197 | 196 |
| 9 | 222 | 201 |
| 10 | 211 | 196 |
| 11 | 187 | 181 |
| 12 | 175 | 164 |
| 13 | 186 | 181 |
| 14 | 243 | 229 |
| 15 | 246 | 231 |

Determine whether the program is effective?

Solution:

**Paired T-Test and CI: Before, After**

Paired T for Before - After

 N Mean StDev SE Mean

Before 15 207.93 28.56 7.37

After 15 197.00 24.39 6.30

Difference 15 10.93 6.33 1.63

95% CI for mean difference: (7.43, 14.44)

T-Test of mean difference = 0 (vs ≠ 0): T-Value = 6.69 P-Value = 0.000

**Example 7:**

A flotoxins produced by mold on peanut crops in Virginia must be monitored. A sample of **64 batches** of peanuts reveals levels of 24.17 ppm, on average, **with a variance of 4.25** ppm. Test the hypothesis that $σ^{2}=4.2$ ppm against the alternative that $σ^{2}\ne 4.2$ ppm.

**Test and CI for One Variance**

Method

Null hypothesis σ-squared = 4.2

Alternative hypothesis σ-squared ≠ 4.2

The chi-square method is only for the normal distribution.

The Bonett method cannot be calculated with summarized data.

Statistics

 N StDev Variance

64 2.06 4.25

95% Confidence Intervals

 CI for CI for

Method StDev Variance

Chi-Square (1.76, 2.50) (3.08, 6.23)

Tests

 Test

Method Statistic DF P-Value

Chi-Square 63.75 63 0.900

Example 8

Hydrocarbon emissions from cars are known to have decreased dramatically during the 1980s. A study was conducted to compare the hydrocarbon emissions at idling speed, in parts per million (ppm), for automobiles from 1980 and 1990. Twenty cars of each model year were randomly selected, and their hydrocarbon emission levels were recorded. The data are as follows:

1980 models:

141 359 247 940 882 494 306 210 105 880

200 223 188 940 241 190 300 435 241 380

1990 models:

140 160 20 20 223 60 20 95 360 70

220 400 217 58 235 380 200 175 85 65

Assume both populations are normal.

1. Test the hypothesis that $σ\_{1}=275$ against the alternative that $σ\_{1}\ne 275$.

**Test and CI for One Variance: 1980 models:**

Method

Null hypothesis σ = 275

Alternative hypothesis σ ≠ 275

The chi-square method is only for the normal distribution.

The Bonett method is for any continuous distribution.

Statistics

Variable N StDev Variance

1980 models: 20 281 78999

95% Confidence Intervals

 CI for CI for

Variable Method StDev Variance

1980 models: Chi-Square (214, 411) (45688, 168525)

 Bonett (195, 449) (37996, 201874)

Tests

 Test

Variable Method Statistic DF P-Value

1980 models: Chi-Square 19.85 19 0.808

 Bonett — — 0.897

1. Test the hypothesis that $σ\_{1}=σ\_{2}$ against the alternative that $σ\_{1}\ne σ\_{2}$.

**Test and CI for Two Variances: 1980 models:, 1990 models:**

Method

Null hypothesis σ(1980 models:) / σ(1990 models:) = 1

Alternative hypothesis σ(1980 models:) / σ(1990 models:) ≠ 1

Significance level α = 0.05

F method was used. This method is accurate for normal data only.

Statistics

 95% CI for

Variable N StDev Variance StDevs

1980 models: 20 281.067 78998.516 (213.749, 410.518)

1990 models: 20 119.395 14255.082 ( 90.798, 174.384)

Ratio of standard deviations = 2.354

Ratio of variances = 5.542

95% Confidence Intervals

 CI for

 CI for StDev Variance

Method Ratio Ratio

F (1.481, 3.742) (2.194, 14.001)

Tests

 Test

Method DF1 DF2 Statistic P-Value

F 19 19 5.54 0.000

**One Way ANOVA**

Suppose in an industrial experiment that an engineer is interested in how the mean absorption of moisture in concrete varies among 5 different concrete aggregates. The samples are exposed to moisture for 48 hours. It is decided that 6 samples are to be tested for each aggregate, requiring a total of 30 samples to be tested. The data are recorded in Table 13.1.

The model for this situation may be set up as follows. There are 6 observations taken from each of 5 populations with means *μ*1*, μ*2*, . . . , μ*5, respectively. We may wish to test

*H*0: *μ*1 = *μ*2 = *· · ·* = *μ*5*,*

*H*1: At least two of the means are not equal*.*

Table 13.1: Absorption of Moisture in Concrete Aggregates

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Aggregate | 1 | 2 | 3 | 4 | 5 |
|  | 551 | 595 | 639 | 417 | 563 |
|  | 457 | 580 | 615 | 449 | 631 |
|  | 450 | 508 | 511 | 517 | 522 |
|  | 731 | 583 | 573 | 438 | 613 |
|  | 499 | 633 | 648 | 415 | 656 |
|  | 632 | 517 | 677 | 555 | 679 |

**One-way ANOVA: t1, t2, t3, t4, t5**

Method

Null hypothesis All means are equal

Alternative hypothesis At least one mean is different

Significance level α = 0.05

Equal variances were assumed for the analysis.

Factor Information

Factor Levels Values

Factor 5 t1, t2, t3, t4, t5

Analysis of Variance

Source DF Adj SS Adj MS F-Value P-Value

Factor 4 85356 21339 4.30 0.009

Error 25 124020 4961

Total 29 209377

Model Summary

 S R-sq R-sq(adj) R-sq(pred)

70.4330 40.77% 31.29% 14.70%

Means

Factor N Mean StDev 95% CI

t1 6 553.3 110.2 (494.1, 612.6)

t2 6 569.3 48.0 (510.1, 628.6)

t3 6 610.5 59.9 (551.3, 669.7)

t4 6 465.2 57.6 (405.9, 524.4)

t5 6 610.7 58.8 (551.4, 669.9)

Pooled StDev = 70.4330

**Tukey Pairwise Comparisons**

Grouping Information Using the Tukey Method and 95% Confidence

Factor N Mean Grouping

t5 6 610.7 A

t3 6 610.5 A

t2 6 569.3 A B

t1 6 553.3 A B

t4 6 465.2 B

Means that do not share a letter are significantly different.

Tukey Simultaneous Tests for Differences of Means

Difference Difference SE of Adjusted

of Levels of Means Difference 95% CI T-Value P-Value

t2 - t1 16.0 40.7 (-103.3, 135.3) 0.39 0.995

t3 - t1 57.2 40.7 ( -62.2, 176.5) 1.41 0.630

t4 - t1 -88.2 40.7 (-207.5, 31.2) -2.17 0.224

t5 - t1 57.3 40.7 ( -62.0, 176.7) 1.41 0.627

t3 - t2 41.2 40.7 ( -78.2, 160.5) 1.01 0.847

t4 - t2 -104.2 40.7 (-223.5, 15.2) -2.56 0.109

t5 - t2 41.3 40.7 ( -78.0, 160.7) 1.02 0.845

t4 - t3 -145.3 40.7 (-264.7, -26.0) -3.57 0.012

t5 - t3 0.2 40.7 (-119.2, 119.5) 0.00 1.000

t5 - t4 145.5 40.7 ( 26.2, 264.8) 3.58 0.012

Individual confidence level = 99.29%

**Tukey Simultaneous 95% CIs**

**Fisher Pairwise Comparisons**

Grouping Information Using the Fisher LSD Method and 95% Confidence

Factor N Mean Grouping

t5 6 610.7 A

t3 6 610.5 A

t2 6 569.3 A

t1 6 553.3 A

t4 6 465.2 B

Means that do not share a letter are significantly different.

Fisher Individual Tests for Differences of Means

Difference Difference SE of Adjusted

of Levels of Means Difference 95% CI T-Value P-Value

t2 - t1 16.0 40.7 ( -67.8, 99.8) 0.39 0.697

t3 - t1 57.2 40.7 ( -26.6, 140.9) 1.41 0.172

t4 - t1 -88.2 40.7 (-171.9, -4.4) -2.17 0.040

t5 - t1 57.3 40.7 ( -26.4, 141.1) 1.41 0.171

t3 - t2 41.2 40.7 ( -42.6, 124.9) 1.01 0.321

t4 - t2 -104.2 40.7 (-187.9, -20.4) -2.56 0.017

t5 - t2 41.3 40.7 ( -42.4, 125.1) 1.02 0.319

t4 - t3 -145.3 40.7 (-229.1, -61.6) -3.57 0.001

t5 - t3 0.2 40.7 ( -83.6, 83.9) 0.00 0.997

t5 - t4 145.5 40.7 ( 61.7, 229.3) 3.58 0.001

Simultaneous confidence level = 73.15%

**Regression Analysis: Expenditure versus Income**

Analysis of Variance

Source DF Seq SS Contribution Adj SS Adj MS F-Value P-Value

Regression 1 7713005 70.63% 7713005 7713005 19.24 0.002

 Income 1 7713005 70.63% 7713005 7713005 19.24 0.002

Error 8 3207268 29.37% 3207268 400908

Total 9 10920272 100.00%

Model Summary

 S R-sq R-sq(adj) PRESS R-sq(pred)

633.173 70.63% 66.96% 4895956 55.17%

Coefficients

Term Coef SE Coef 99% CI T-Value P-Value VIF

Constant -495 839 (-3312, 2322) -0.59 0.572

Income 0.723 0.165 (0.170, 1.275) 4.39 0.002 1.00

Regression Equation

Expenditure = -495 + 0.723 Income

**Correlation: Expenditure, Income**

Pearson correlation of Expenditure and Income = 0.840

P-Value = 0.002

**Tabulated Statistics: R, C**

Using frequencies in C1

Rows: R Columns: C

 High

 Bechelors School Masters PhD All

F 54 60 46 41 201

 49.87 50.89 50.38 49.87

M 44 40 53 57 194

 48.13 49.11 48.62 48.13

All 98 100 99 98 395

Cell Contents: Count

 Expected count

Pearson Chi-Square = 8.006, DF = 3, P-Value = 0.046

Likelihood Ratio Chi-Square = 8.045, DF = 3, P-Value = 0.045