

**Example (1)**

You are a financial analyst for a brokerage firm. Is there a difference in dividend yield between stocks listed on the NYSE & NASDAQ? You collect the following data:

$$\text{NYSE: } n_1 = 21, \quad \bar{X}_1 = 3.27, \quad S_1 = 1.30$$

$$\text{NASDAQ: } n_2 = 25, \quad \bar{X}_2 = 2.53, \quad S_2 = 1.16$$

Assuming both populations are approximately normal with equal variances, is there a difference in mean yield ( $\alpha = 0.05$ )?

**Solution:**

( $\sigma_1$  &  $\sigma_2$  unknown) ( $\sigma_1 = \sigma_2$ ) t

**Step 1: state the hypothesis:**

$$H_0: \mu_1 = \mu_2$$

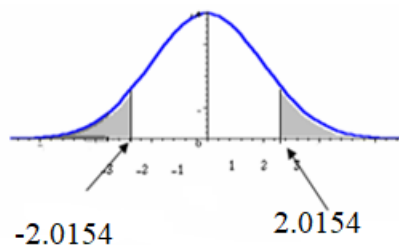
$$H_1: \mu_1 \neq \mu_2$$

Test is two-tailed test (key word is difference between 2 samples)

**Step 2- Select the level of significance and critical value.**

$\alpha = 0.05$  as stated in the problem

$$\pm t\left(\frac{\alpha}{2}, n_1 + n_2 - 2\right) = \pm t\left(\frac{0.05}{2}, 21 + 25 - 2\right) = \pm t(0.025, 44) = \pm 2.0154$$



**Step 3: Select the test statistic.**

Use Z-distribution since the assumptions are met

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(21 - 1) \times 1.3^2 + (25 - 1) \times 1.16^2}{(21 - 1) + (25 - 1)}$$
$$= \frac{33.8 + 32.2944}{44} = \frac{66.0944}{44} = 1.5021$$

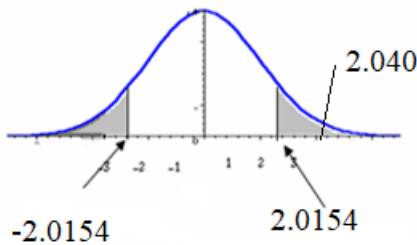
$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$t_{stat} = \frac{3.27 - 2.53}{\sqrt{1.5021 \left( \frac{1}{21} + \frac{1}{25} \right)}} = \frac{0.74}{0.3627} = 2.040$$

**Step 4: Formulate the decision rule. (Critical value)**

Reject  $H_0$  if  $t_c > 2.0154$  Or  $t_c < -2.0154$

**Step 5: Make a decision and interpret the result.**



$$\begin{array}{l} \times \quad H_0: \mu_1 \leq \mu_2 \\ \checkmark \quad H_1: \mu_1 > \mu_2 \end{array}$$

Reject  $H_0$  at  $\alpha = 0.05$ , There is evidence of a difference in means.

*Reject  $H_0$ . There is sufficient evidence that  $\mu_{NYSE} > \mu_{NASDAQ}$*

95% Confidence Interval for  $\mu_{\text{NYSE}} - \mu_{\text{NASDAQ}}$

$$(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} \sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} = 0.74 \pm 2.0154 \times 0.3628 = (0.009, 1.471)$$

Since 0 is less than the entire interval, we rejected  $H_0$  and we can be 95% confident that  $\mu_{\text{NYSE}} > \mu_{\text{NASDAQ}}$

## Example (2)

Assume you send your salespeople to a “customer service” training workshop. Has the training made a difference in the number of complaints (at the 0.01 level)? You collect the following data:

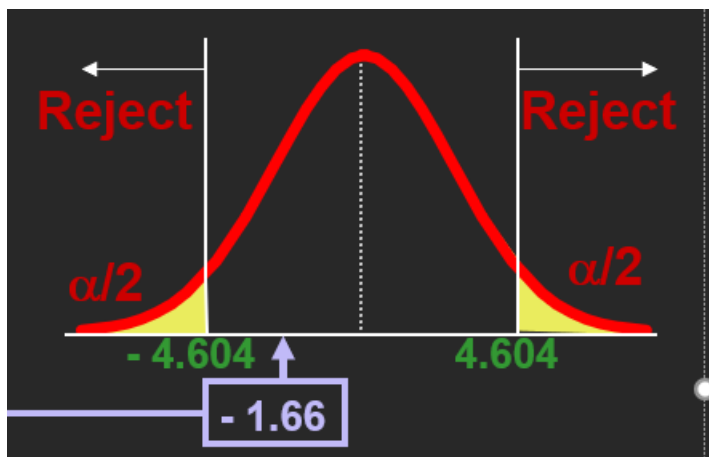
<u>Salesperson</u>	<u>Number of Complaints</u>	<u>Number of Complaints</u>
	Before	After
C.B.	6	4
T.F.	20	6
M.H.	3	2
R.K.	0	0
M.O.	4	0

**Solution:**

**Step 1: state the hypothesis:**

$$H_0: \mu_D = 0$$

$$H_1: \mu_D \neq 0$$



**Step2: Select the level of significance and critical value.**

$$t_{0.005} = \pm 4.604 \quad \text{d.f.} = n - 1 = 4$$

**Step 3: Find the appropriate test statistic.**

<u>Salesperso</u> <u>n</u>	<u>Number of</u> <u>Complaints</u> Before	<u>Number of</u> <u>Complaints</u> After	$D$ (X2-X1)	$(D_i - \bar{D})$	$(D_i - \bar{D})^2$
<b>C.B.</b>	6	4	-2	-2-(-4.2)= 2.2	4.84
<b>T.F.</b>	20	6	-14	-14-(-4.2)=-9.8	96.04
<b>M.H.</b>	3	2	-1	-1-(-4.2)= 3.2	10.24
<b>R.K.</b>	0	0	0	0-(-4.2)=4.2	17.64
<b>M.O.</b>	4	0	-4	4-(-4.2)=0.2	0.04
Total			-21		128.8

$$\bar{D} = \frac{\sum D_i}{n} = \frac{-21}{5} = -4.2$$

$$S_D = \sqrt{\frac{\sum (D_i - \bar{D})^2}{n - 1}} = \sqrt{\frac{128.8}{4}} = 5.6745$$

$$t_c = \frac{\bar{D}}{S_D / \sqrt{n}} = \frac{-4.2}{5.6745 / 2.2361} = \frac{-4.2}{2.5377} = -1.66$$

#### Step 4: State the decision rule

Reject  $H_0$  if

$$t_c > 4.604$$

or

$$t_c < -4.604$$

#### Step 5: Decision Reject $H_0$

$$\begin{array}{l} \checkmark \quad H_0: \mu_D = 0 \\ \times \quad H_1: \mu_D \neq 0 \end{array}$$

Do not reject  $H_0$  ( $t_{\text{stat}}$  is not in the rejection region)

Do not reject  $H_0$ . There is insufficient of a change in the number of complaints.

$$\mu_{\text{complaints before training workshop}} = \mu_{\text{complaints after training workshop}}$$

**-The Paired Difference Confidence Interval  $\mu_D$  is:**

$$\begin{aligned} \hat{\mu}_D &= \bar{D} \pm t_{\alpha/2} \frac{S_D}{\sqrt{n}} \\ &= -4.2 \pm 4.604 \frac{5.6745}{\sqrt{5}} \\ &= -4.2 \pm 11.6836 \\ -15.87 &< \hat{\mu}_D < 7.48 \end{aligned}$$

Since this interval contains 0 you are 99% confident that  $\mu_D = 0$

Do not reject  $H_0$

$$\mu_{\text{complaints before training workshop}} = \mu_{\text{complaints after training workshop}}$$

### Example (3)

Is there a significant difference between the proportion of men and the proportion of women who will vote Yes on Proposition A?

In a random sample, 36 of 72 men and 35 of 50 women indicated they would vote Yes

Test at the .05 level of significance

**Solution:**

**Step 1: State the null and alternate hypotheses.**

$H_0: \pi_1 - \pi_2 = 0$  (the two proportions are equal)

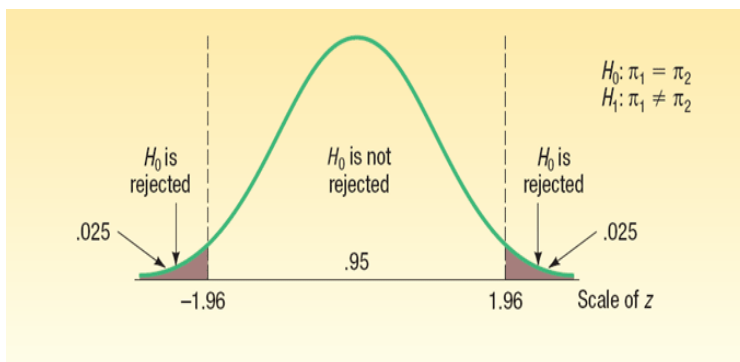
$H_1: \pi_1 - \pi_2 \neq 0$  (there is a significant difference between proportions)

**Step 2: State the level of significance and critical value.**

The .05 significance level is stated in the problem.

$$\pm Z_{\frac{\alpha}{2}} = \pm Z_{\frac{0.05}{2}} = \pm Z_{0.025}$$

$$\pm Z_{0.025} = \pm 1.96$$



### Step 3: Find the appropriate test statistic.

The sample proportions are:

$$\text{Men: } p_1 = 36/72 = 0.50$$

$$\text{Women: } p_2 = 35/50 = 0.70$$

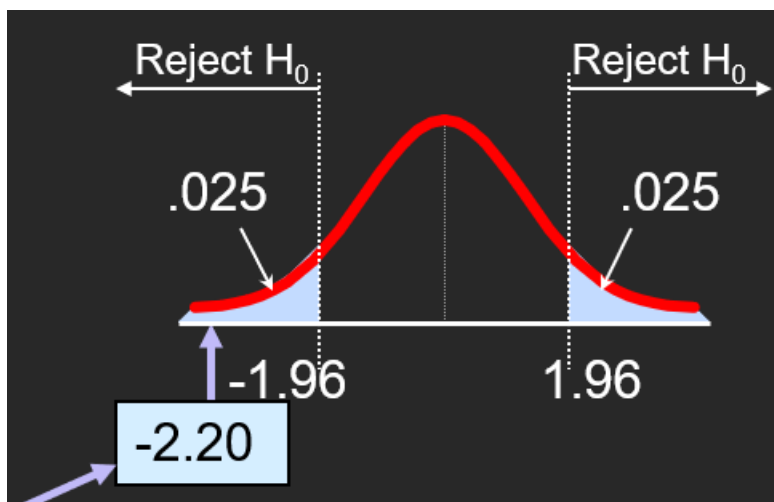
The pooled estimate for the overall proportion is:

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{36 + 35}{72 + 50} = \frac{71}{122} = .582$$

$$\begin{aligned} z_{\text{STAT}} &= \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1 - \bar{p}) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} \\ &= \frac{(.50 - .70) - (0)}{\sqrt{.582(1 - .582) \left( \frac{1}{72} + \frac{1}{50} \right)}} = -2.20 \end{aligned}$$

### Step 4: State the decision rule

$$\begin{aligned} \text{Reject } H_0 \text{ if } & z_c > Z_{\frac{\alpha}{2}} \quad \text{Or} \quad z_c < -Z_{\frac{\alpha}{2}} \\ & z_c > 1.96 \quad \text{Or} \quad z_c < -1.96 \end{aligned}$$





### Step 5: Decision Reject $H_0$

There is evidence of a significant **difference** in the proportion of men and women who will vote yes.

$$\begin{array}{l} \times \quad H_0: \pi_1 \leq \pi_2 \\ \checkmark \quad H_1: \pi_1 > \pi_2 \end{array}$$

*Reject  $H_0$ . There is sufficient evidence that  $\pi_{weman} > \pi_{men}$*

The confidence interval for

$\pi_1 - \pi_2$  is:

$$(p_1 - p_2) \pm Z_{\alpha/2} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

The 95% confidence interval for  $\pi_1 - \pi_2$  is:

$$\begin{aligned} (0.50 - 0.70) \pm 1.96 \sqrt{\frac{0.50(0.50)}{72} + \frac{0.70(0.30)}{50}} \\ = (-0.37, -0.03) \end{aligned}$$

*Since this interval does not contain 0 can be 95% confident the two proportions are different. There is sufficient evidence that  $\pi_{weman} > \pi_{men}$*

### Example (4)

You are a financial analyst for a brokerage firm. Is there a difference in dividend yield between stocks listed on the NYSE & NASDAQ? You collect the following data:

NYSE:  $n_1 = 21$  ,  $S_1 = 1.30$

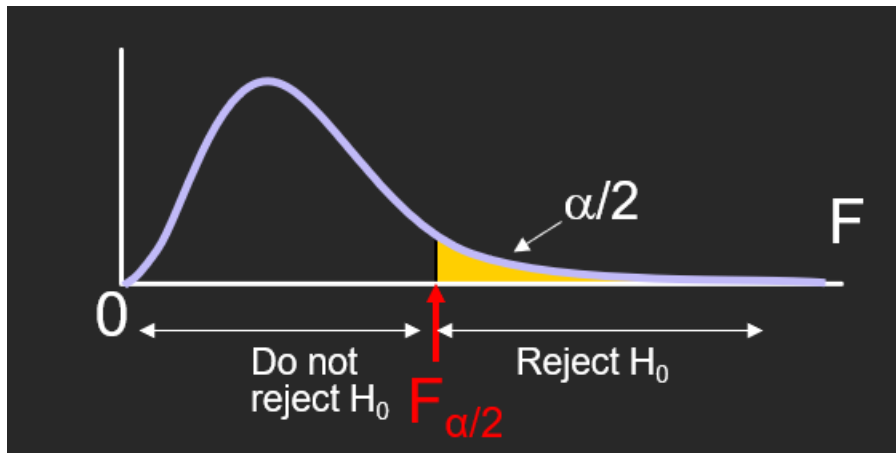
NASDAQ:  $n_2 = 25$  ,  $S_2 = 1.16$

Is there a difference in the variances between the NYSE & NASDAQ at the  $\alpha = 0.05$  level?

**Step 1: state the hypothesis:**

$H_0: \sigma^2_1 = \sigma^2_2$  (there is no difference between variances)

$H_1: \sigma^2_1 \neq \sigma^2_2$  (there is a difference between variances)



**Step2: Select the level of significance and critical value.**

Find the F critical value for  $\alpha = 0.05$ :

Numerator d.f. =  $n_1 - 1 = 21 - 1 = 20$

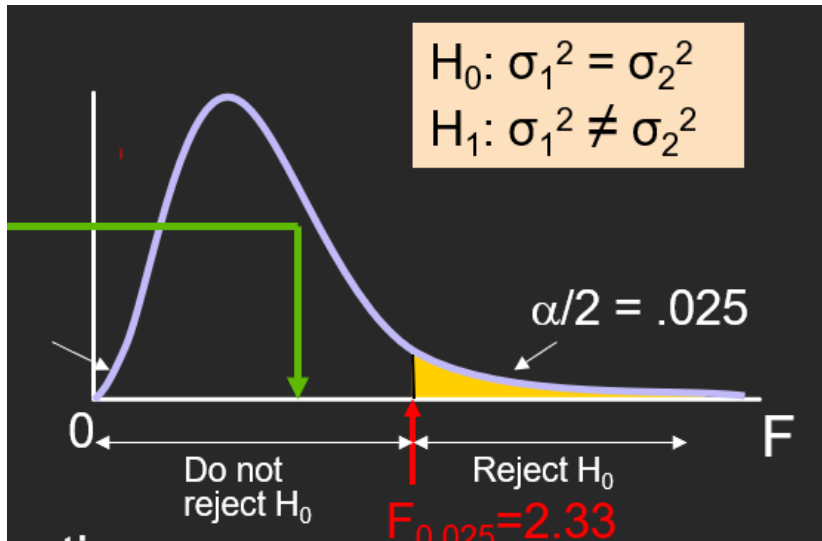
Denominator d.f. =  $n_2 - 1 = 25 - 1 = 24$

$$F_{\alpha/2} = F_{.025, 20, 24} = 2.33$$

### Step 3: Find the appropriate test statistic.

The test statistic is:

$$F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{1.30^2}{1.16^2} = 1.256$$



### Step 4: State the decision rule

**Reject  $H_0$  if  $F_{STAT} > F_{\alpha/2} = 2.33$**

$F_{STAT} = 1.256$  is not in the rejection region, so we do not reject  $H_0$

### Step 5: Decision Reject $H_0$

because  $F_{stat} = 1.256 < 2.33$ , you do not reject  $H_0$ . using a 0.05 level of significance, you conclude that there is evidence that there is no variability in the NYSE & NASDAQ.

Do not reject the null hypothesis, there was no significant difference between two variances ( $\sigma_1^2 = \sigma_2^2$ ). The test statistic will be use is Pooled variance t-test

✓  $H_0: \sigma_1^2 = \sigma_2^2$  (there is no difference between variances)

×  $H_1: \sigma_1^2 \neq \sigma_2^2$  (there is a difference between variances)

Cumulative Probabilities = 0.975													
Upper-Tail Areas = 0.025													
Numerator, $df_1$													
Denominator, $df_2$	1	2	3	4	5	6	7	8	9	10	12	15	20
1	647.80	799.50	864.20	899.60	921.80	937.10	948.20	956.70	963.30	968.60	976.70	984.90	993.10
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.39	39.39	39.40	39.41	39.43	39.45
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33

### Example (5) Text book pages (372 & 373)

Waiting time is a critical issue at fast-food chains, which not only want to minimize the mean service time but also want to minimize the variation in the service time from customer to customer. One fast-food chain carried out a study to measure the variability in the waiting time (defined as the time in minutes from when an order was completed to when it was delivered to the customer) at lunch and breakfast at one of the chain's stores. The results were as follows:

Lunch:  $n_1 = 25$  ,  $S_1^2 = 4.4$

Breakfast:  $n_2 = 21$  ,  $S_2^2 = 1.9$

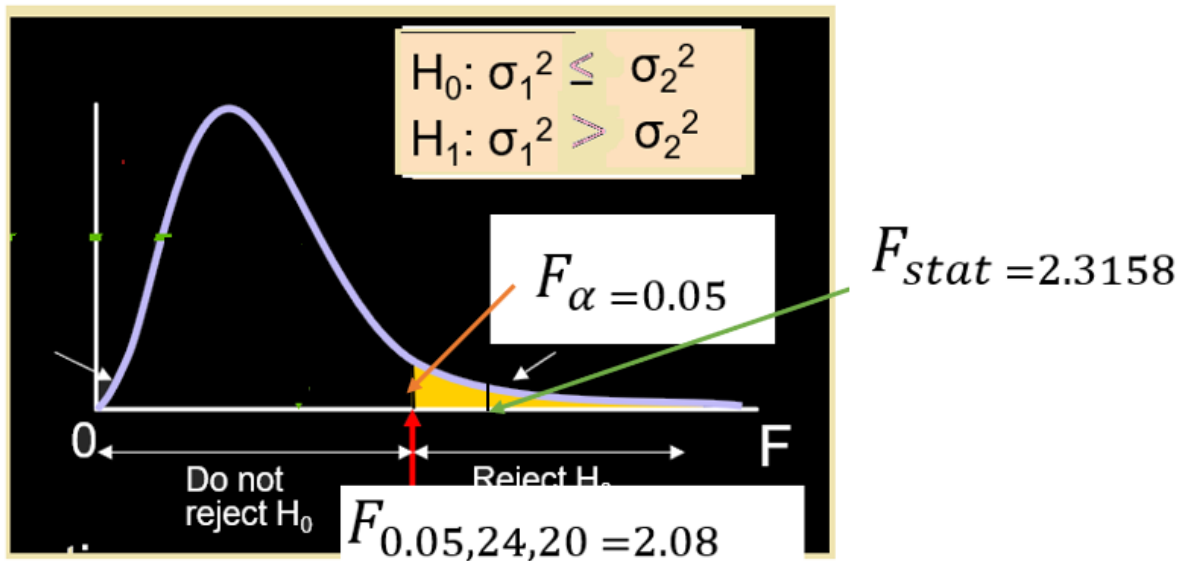
At the 0.05 level of significance, is there evidence that there is more variability in the service time at lunch than at breakfast? Assume that the population service times are normally distributed

Solution

1)

$$H_0: \sigma_1^2 \leq \sigma_2^2$$

$$H_1: \sigma_1^2 > \sigma_2^2$$



2)

$$F_{\alpha, n_1-1, n_2-1} = F_{0.05, 25-1, 21-1} = F_{0.05, 24, 20} = 2.08$$

3)

$$F_{stat} = \frac{S_1^2}{S_2^2} = \frac{4.4}{1.9} = 2.3158$$

4)

Reject  $H_0$  if  $F_{stat} > 2.08$

5) because  $F_{stat} = 2.3158 > 2.08$ , you reject  $H_0$ . using a 0.05 level of significance, you conclude that there is evidence that there is more variability in the service time at lunch than at breakfast.

Reject the null hypothesis, there was a significant difference between two variances ( $\sigma_1^2 \neq \sigma_2^2$ ). The test statistic will be use is Separate variance t-test

$$\begin{array}{l} \times \\ \sqrt \end{array} \quad \begin{array}{l} H_0: \sigma_1^2 \leq \sigma_2^2 \\ H_1: \sigma_1^2 > \sigma_2^2 \end{array}$$

Upper-Tail Areas = 0.05														
Numerator, $df_1$														
Denominator, $df_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24
1	161.40	199.50	215.70	224.60	230.20	234.00	236.80	238.90	240.50	241.90	243.90	245.90	248.00	249.10
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.38	19.40	19.41	19.43	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08
21	4.32	3.46	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.17	2.10	2.05

**Example (6)** An experiment has a single factor with four groups and nine values in each group.

If  $SSA=752$ ,  $SST=1250$

Answer the following questions:

- 1) How many degrees of freedom are there in determining the among-group variation?
- 2) How many degrees of freedom are there in determining the within-group variation?
- 3) How many degrees of freedom are there in determining the total variation?
- 4) What is  $SSW$ ?
- 5) What is  $MSA$ ?
- 6) What is  $MSW$ ?
- 7) What is the value of  $F_{stat}$  ?

Solution:

1) How many degrees of freedom are there in determining The among-group variation?

$$c-1=4-1=3$$

2) How many degrees of freedom are there in determining The within-group variation?

$$n = 4 \times 9 = 36$$

$$n-c=36-4=32$$

3) How many degrees of freedom are there in determining The total variation?

$$n-1=36-1=35$$

4)What is  $SSW$ ?

$$SST-SSA=1250-752=498$$

5)What is MSA?

$$MSA = \frac{752}{3} = 250.67$$

6)What is MSW?

$$MSW = \frac{498}{32} = 15.5625$$

7)What is the value of  $F_{stat}$  ?

$$F_{stat} = \frac{MSA}{MSW} = \frac{250.67}{15.5625} = 16.11$$



**Example (7) (Slide 65-68)**

You want to see if three different golf clubs yield different distances. You randomly select five measurements from trials on an automated driving machine for each club. At the 0.05 significance level, is there a difference in mean distance?

	Club 1	Club 2	Club 3
	254	234	200
	263	218	222
	241	235	197
	237	227	206
	251	216	204
Total	1246	1130	1029
Mean	249.2	226	205.8
$\bar{X} = 227$			

C=3

n=15

SSA = 4716.4 , SSW = 1119.6

Solution:

Source of variation (S.V)	Degrees of freedom	Sum of Squares (S.S)	Mean Squares (MS)	F- ratio
Among groups	c-1=3-1=2	SSA=4716.4	MSA =SSA/c-1 $\frac{4716.4}{2}$ = 2358.2	F <sub>STAT</sub> = MSA / MSW  = $\frac{2358.2}{93.3}$ = 25.275
Within groups	n-c=15-3=12	SSW=1119.6	MSW = SSW/n-c $\frac{1119.6}{12}$ = 93.3	
Total	n-1=15-1=14	SST=5836		

**Step (1) : State the null and alternate hypotheses :**

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

$H_1$  : Not all  $\mu_j$  are equal(Not all the means are equal.).

**Step (2): Select the level of significance ( $\alpha = 0.05$ )**

**Step (3): The test statistic :**

$$F_{STAT} = \frac{MSA}{MSW} = \frac{SSA/c - 1}{SSW/n - c} = \frac{4716.4/3 - 1}{1119.6/15 - 3} = 25.275$$

**Step (4): The critical value:**

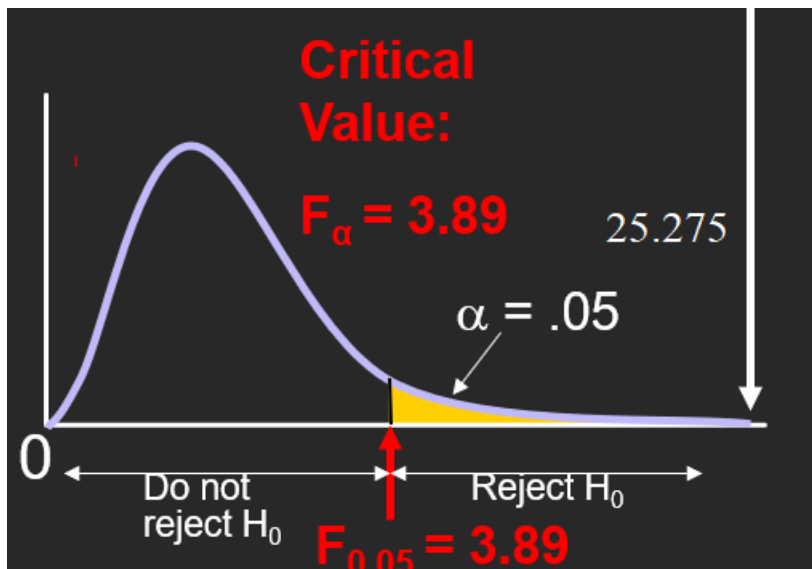
The degrees of freedom for the numerator (c-1) = 3-1 =2

The degrees of freedom for the denominator (n-c) = 15-3 =12

$$F_{(0.05, 2, 12)} = 3.89$$

**Step (5) : Formulate the decision Rule and make a decision**

$$F_{STAT} (25.275) > F_{(0.05, 2, 12)} (3.89)$$



Reject  $H_0$  at  $\alpha = 0.05$

There is evidence that at least one  $\mu_j$  differs from the rest

Since  $F_{stat} > 3.89$  , we reject  $H_0$ . And there is sufficient evidence that at least one  $\mu_j$  differs .

Cumulative Probabilities = 0.95											
Upper-Tail Areas = 0.05											
Numerator, $df_1$											
Denominator, $df_2$	1	2	3	4	5	6	7	8	9	10	12
1	161.40	99.50	215.70	224.60	230.20	234.00	236.80	238.90	240.50	241.90	243.90
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60

### Example (8) Text book page 386

Consider an experiment with four groups, with eight values in each. For the ANOVA summary table below, fill in all the missing results:

#### ANOVA

Sources of variation	Degree of freedom	Sum of squares	Mean squares MS.	F-ratio
Among groups	$c-1=?$	$SSA=?$	$MSA=80$	$F_{STAT}=?$
Within groups	$n-c=?$	$SSW=560$	$MSW=?$	
Total	$n-1=?$	$SST=?$		

a) At the 0.05 level of significance, state the decision rule for testing the null hypothesis that all four groups have equal population means.

b) What is your statistical decision?

Solution:

$$\text{Step (1) } c-1=4-1=3, \quad n=4 \times 8=32, \quad n-c=32-4=28, \quad n-1=32-1=31$$

$$\text{Step (2) } SSA = MSA \times (c - 1) = 80 \times 3 = 240$$

$$\text{Step (3) } MSW = \frac{560}{28} = 20$$

$$\text{Step (4) } SST = SSA + SSW = 240 + 560 = 800$$

$$\text{Step (5) } F_{stat} = \frac{MSA}{MSW} = \frac{80}{20} = 4$$

Sources of variation	Degree of freedom	Sum of squares	Mean squares MS.	F-ratio
Among groups	3	SSA= 240	MSA=80	$F_{STAT} = 4$
Within groups	28	SSW=560	MSW=20	
Total	31	SST= 800		

a) At the 0.05 level of significance, state the decision rule for testing the null hypothesis that all four groups have equal population means.

*Reject  $H_0$  if  $F_{stat} > 2.95$ ; otherwise do not reject  $H_0$*

b) What is your statistical decision?

*Because  $F_{stat} = 4 > 2.95$ , reject  $H_0$*

Cumulative Probabilities = 0.95											
Upper-Tail Areas = 0.05											
Numerator, $df_1$											
Denominator, $df_2$	1	2	3	4	5	6	7	8	9	10	12
1	161.40	199.50	215.70	224.60	230.20	234.00	236.80	238.90	240.50	241.90	243.90
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10