King Saud University
Department of Mathematics
2nd Semester 1432-1433 H

MATH 244 (Linear Algebra)
Final Exam
Duration: 3 Hours

Student's Name	Student's ID	Group No.	Lecturer's Name

Question No.	I	II	III	IV	Total
Mark					

[I] **A.** Determine whether the following is **True** or **False**. **[7 Points]**

(1) The matrix AA^T is symmetric, where A is an $n \times n$ matrix. ( )

(2) If $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r\}$ is a linearly independent set of vectors in a vector space V, then $r > \dim(V)$. ( )

(3) If $S = \{\mathbf{u}, \mathbf{u} + \mathbf{v}\}$, then $\mathbf{u} - \mathbf{v} \in Span(S)$. ( )

(4) In the vector space $V = \mathbb{R}^+$ (the positive real numbers) together with the operations $\mathbf{u} + \mathbf{v} = \mathbf{uv}$ and $k\mathbf{u} = \mathbf{u}^k$, the zero vector equals 1. ( )

(5) If $(\mathbf{v})_S = (1, 0, 1)$ for the basis $S = \{(1, 0, 0), (2, 2, 0), (3, 3, 3)\}$ of $\mathbf{R}^3$, then $\mathbf{v} = (4, 3, 3)$. ( )

(6) If A is a 3×5 matrix, then $nullity(A)$ can be $2, 3, 4$ or 5. ( )

(7) $\{1 + x, 1 - x, x^2\}$ is a basis for P_2. ( )

OVER

[I] **B.** Choose the correct answer. **[11 Points]**

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(1) If $A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$, then $A\,(adj(A))$ is

(a) I_2 **(b)** $2I_2$ **(c)** $-4I_2$ **(d)** None of the previous

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(2) The angle θ between $\mathbf{u} = (1,1)$ and $\mathbf{v} = (2,0)$ equals

(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{3}$ **(c)** $\frac{\pi}{4}$ **(d)** None of the previous

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(3) The standard matrix of a counterclockwise rotation on $\mathbb{R}^3$ about the positive x-axis through an angle θ is

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ **(b)** $\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ **(c)** $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **(d)** None of the previous

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(4) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$, then $\begin{vmatrix} 2g & d & a \\ 2h & e & b \\ 2i & f & c \end{vmatrix}$ equals

(a) -8 **(b)** 4 **(c)** 8 **(d)** None of the previous

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(5) The eigenvalues of A^3 where $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & -8 \\ 0 & 0 & 2 \end{bmatrix}$ are

(a)1,-1,8 **(b)** 1,-1,2 **(c)** 1,4 **(d)** None of the previous

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(6) If $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $T_1(x,y) = (2x+y, x+3y)$ and $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $T_2(x,y) = (x,-y)$, then $[T_2 \circ T_1]$ is

(a) $\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix}$ **(c)** $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ **(d)** None of the previous

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(7) Let $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ be the multiplication by $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 3 \\ 2 & -2 & 4 \end{bmatrix}$. For $\mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $T_A(5e_3)$ equals

(a) $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ **(b)** $\begin{bmatrix} 10 \\ 15 \\ 20 \end{bmatrix}$ **(c)** $\begin{bmatrix} 250 \\ 375 \\ 500 \end{bmatrix}$ **(d)** None of the previous

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(8) If $S = \{(1,-1,0,1), (-1,1,1,0), (2,-2,1,3)\}$ and $W = Span(S)$, then $\dim(W)$ is

(a) 1 **(b)** 2 **(c)** 3 **(d)** None of the previous

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(9) If A is a 5×7 matrix with $rank(A) = 4$, then the number of parameters in the general solution of the linear system $A\mathbf{x} = \mathbf{0}$ is

(a) 3 **(b)** 4 **(c)** 5 **(d)** None of the previous

OVER

[II] A. Let $\mathbf{v}_1 = (1, 0, 1, 1), \mathbf{v}_2 = (-3, 3, 7, 1), \mathbf{v}_3 = (-1, 3, 9, 3)$ and $\mathbf{v}_4 = (-5, 3, 5, -1)$ be vectors in $\mathbb{R}^4$. **[8 Points]**

(i) Find a subset of $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ that forms a basis for the space $W = Span(S)$

(ii) Express each vector not in the basis as a linear combination of the basis vectors.

OVER

[II] B. Suppose that $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ is a matrix transformation with standard matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. **[8 Points]**

(i) Show that T_A is one-to-one.

(ii) Find $\left[T_A^{-1}\right]$

(iii) Evaluate $T_A^{-1}(1, 2, 3)$

OVER

[III] A. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. **[5 Points]**

(i) Compute the eigenvalues of A.

(ii) Determine whether A is invertible or not.

[III] B. Suppose that $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. **[6 Points]**

(i) Show that $\lambda = 2$ is an eigenvalue of A.

(ii) Find a basis for the eigenspace of A corresponding to $\lambda = 2$.

OVER

[V] A. For $A = \begin{bmatrix} -1 & 1 & -2 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$, evaluate **[5 Points]**

(i) $rank(A)$

(ii) $nullity(A^T)$

[V] B.(BONUS) **[5 Points]**

(i) Show that $W = \{A \in M_{nn} : nullity(A) = 0\}$ is not a subspace of M_{nn}.

(ii) Find the conditions that must be satisfied by b_1, b_2, b_3 and b_4 for the following overdetermined system to be consistent.

$$\begin{aligned} x_1 + x_2 &= b_1 \\ x_1 + 2x_2 &= b_2 \\ x_1 - x_2 &= b_3 \\ 2x_1 + 4x_2 &= b_4 \end{aligned}$$

Good Luck